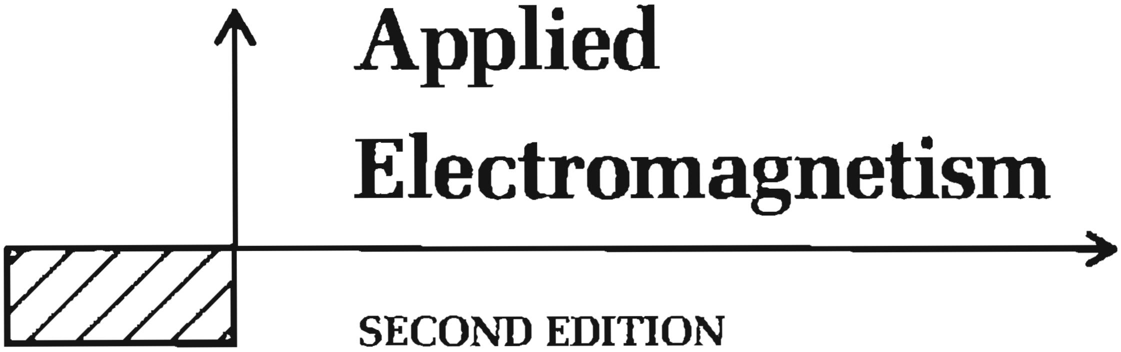
Solutions Manual for



Applied Electromagnetism

SECOND EDITION

Shen

Huang

Solutions Manual

for

Shen and Kong's

APPLIED ELECTROMAGNETISM

Second Edition

by

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PWS-KENT Publishing Company
Boston

PWS-KENT
Publishing Company

20 Park Plaza
Boston, Massachusetts 02116

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PWS-KENT Publishing Company is a division of Wadsworth, Inc.

ISBN 0-534-07621-1

Printed in the United States of America

91 — 10 9 8 7 6 5 4

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Notice that $\mathbf{A}\cdot\mathbf{B}^* = -2j$ and that $\langle A(t)\cdot B(t)\rangle = (1/2)\mathrm{Re}\{\mathbf{A}\cdot\mathbf{B}^*\} = 0$. We should also note that $\mathbf{A}\cdot\mathbf{B} = 0$ and that $A(t)\cdot B(t) = 0$. Thus, the two time-domain vectors are always perpendicular to each other.

Example 1.20

Consider the two vectors $\mathbf{A} = \hat{x} + j\hat{y}$ and $\mathbf{B} = \hat{x} + j\hat{y}$. (They are actually the same vector.) We find that $\mathbf{A}\times\mathbf{B} = 0$ and that $\mathbf{A}\cdot\mathbf{B} = 0$. Are the two vectors parallel to each other or perpendicular to each other?

Solution: Consider instead $\mathbf{A}\times\mathbf{B}^*$ and $\mathbf{A}\cdot\mathbf{B}^*$. We find that $\mathbf{A}\times\mathbf{B}^* = -\hat{z}2j$ and that $\mathbf{A}\cdot\mathbf{B}^* = 2$. Thus, $\langle A(t)\times B(t)\rangle = 0$, and that $\langle A(t)\cdot B(t)\rangle = 1$. Furthermore, $A(t)\times B(t) = 0$, and $A(t)\cdot B(t) = 1$. Thus, the time-domain vector is parallel to itself all the time.

Problems

1.1 Let $a = 8 + j2$ and $b = -3 + j$. Calculate (a) $a + b$, (b) $a - b$, (c) ab, and (d) a/b. Give the answer in real and imaginary parts.

1.2 Repeat (c) and (d) in Problem 1.1 with the answer given in phasor form.

1.3 Find the real part, the imaginary part, and the magnitude of $e^{j\omega t}$, where ω and t are real numbers.

·1.4 Let c be a complex number. Are the following statements always true?

(a) $(c + c^*)$ is real.
(b) $(c - c^*)$ is imaginary.
(c) c/c^* has a magnitude equal to 1.

1.5 Consider the equation $z^2 = 1 + j$. Find two values of z that satisfy this equation.

1.6 Let a be a real number, and let $|a| \ll 1$. Show that the square root of $(1 + ja)$ is approximately equal to $\pm(1 + ja/2)$.

1.7 Let a be a positive real number, and let $a \gg 1$. Show that the square root of $(1 + ja)$ is approximately equal to $\pm(1 + j)(a/2)^{1/2}$.

·1.8 Obtain the phasor notation of the following time-harmonic functions (if possible):

(a) $V(t) = 6\cos(\omega t + \pi/4)$
(b) $I(t) = -8\sin(\omega t)$
(c) $A(t) = 3\sin(\omega t) - 2\cos(\omega t)$
(d) $C(t) = 6\cos(120\pi t - \pi/2)$
(e) $D(t) = 1 - \cos(\omega t)$
(f) $U(t) = \sin(\omega t + \pi/3)\sin(\omega t + \pi/6)$

1.9 Obtain $C(t)$ in terms of ω from the following phasors: (a) $C = 1 + j$, (b) $C = 4\exp(j0.8)$, and (c) $C = 3\exp(j\pi/2) + 4\exp(j0.8)$.

·1.10 Show that, if $V = r + jx$ and $U = g + jy$, then $V(t)U(t) \neq \mathrm{Re}\{VU\,e^{j\omega t}\}$. Find the correct expression for $V(t)U(t)$ in terms of r, x, g, y, and ωt.

1.11 Let $A = -8\hat{x} + 9\hat{y} - \hat{z}$ and $B = 2\hat{x} - 4\hat{y} + 3\hat{z}$. Find (a) $A + B$, (b) $A - B$, (c) $A \cdot B$, and (d) $A \times B$.

1.12 Find the angle between A and B that are given in Problem 1.11.

1.13 Show that for

$$V(t) = V_0 \cos(\omega t + \phi) = \text{Re}\{\mathrm{V}\, e^{j\omega t}\}$$

$$\frac{\partial}{\partial t} V(t) \leftrightarrow j\omega \mathrm{V}$$

1.14 Find a vector **C** that is perpendicular to $A = -8\hat{x} + 9\hat{y} - \hat{z}$, has no $\hat{z}$ component, and has a magnitude equal to 1.

1.15 Find the vector C that is parallel to $A = 5\hat{x} - 8\hat{y} + 2\hat{z}$ and has a magnitude equal to 1.

1.16 Find a unit vector $\hat{n}$ that points in the same direction as an arrow drawn from point A to point B where the rectangular coordinates of A and B are (1,0,2) and (−1,3,−2), respectively.

1.17 Show that the definition of the dot product $V \cdot U$ given by (1.10a) is equivalent to that given by (1.10b). To simplify the algebra, you may choose the coordinates so that the x axis is along V and the z axis is perpendicular to both V and U. In other words, let $V = a\hat{x}$ and $U = b\hat{x} + c\hat{y}$.

1.18 Prove (1.11b) using the approach suggested in the text.

1.19 Show that the definition of the cross product $V \times U$ given by (1.12) is equivalent to that given by (1.14). To simplify the algebra, do what is suggested in Problem 1.17.

1.20 Figure P1.20 shows that a vector V is along the x axis and a vector U is on the x-y plane forming a 135° angle with the x axis. The magnitudes of these vectors are v and u, respectively. Use (1.14) to express $V \times U$ in terms v and u. Now, we can also say that the angle between V and U is 225°. Use (1.14) and explain why the vector $V \times U$ is the same whether or not we choose the angle between them to be 135° or 225°. Hint: Follow the right-hand rule. In determining the direction vector $\hat{n}$, your fingers must always point from V to U in the direction you measure the angle between them.

1.21 Prove (1.15) using the approach suggested in the text.

1.22 Find the phasor notations of the following time-harmonic vectors:
(a) $V(t) = 3\cos(\omega t)\hat{x} + 4\sin(\omega t)\hat{y} + \hat{z}\cos(\omega t + \pi/2)$
(b) $E(t) = [3\cos(\omega t) + 4\sin(\omega t)]\hat{x} + 8[\cos(\omega t) - \sin(\omega t)]\hat{z}$
(c) $H(t) = 0.5\cos(kz - \omega t)\hat{x}$

1.23 Find the phasor notation of the following vector:

$$C(z,t) = (\partial/\partial t)\, E(z,t)$$

where E is given in Problem 1.22(b).

1.24 From the following complex vectors, find **C**(t) in terms of ωt: (a) $\mathbf{C} = \hat{x} - j\hat{y}$, (b) $\mathbf{C} = j(\hat{x} - j\hat{y})$, and (c) $\mathbf{C} = \exp(-jkz)\hat{x} + j\exp(jkz)\hat{y}$.

1.25 Let $\mathbf{A} = \hat{\mathbf{x}} + j\hat{\mathbf{y}} + (1 + j2)\hat{\mathbf{z}}$, and let $\mathbf{B} = -\hat{\mathbf{x}} - (1 - j2)\hat{\mathbf{y}} + j\hat{\mathbf{z}}$. Find (a) $\mathbf{A} + \mathbf{B}$, (b) $\mathbf{A} - \mathbf{B}$, (c) $\mathbf{A} \cdot \mathbf{B}$, and (d) $\mathbf{A} \times \mathbf{B}$.

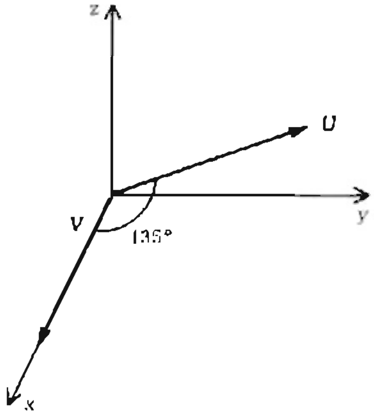


Figure P1.20

1.26 Find $\mathbf{A} \cdot \mathbf{A}^*$ and $\text{Re}\{\mathbf{A} \times \mathbf{B}^*\}$ for the values of $\mathbf{A}$ and $\mathbf{B}$ given in Problem 1.25.

1.27 Sketch the trace of the tip of the vector $\mathcal{A}(t)$, where (a) $\mathbf{A} = \hat{\mathbf{x}} - j\hat{\mathbf{y}}$ and where (b) $\mathbf{A} = 4\hat{\mathbf{x}} + j3\hat{\mathbf{y}}$.

1.28 Calculate $\mathbf{A} \cdot \mathbf{B}$, given $\mathbf{A} = \hat{\mathbf{x}} + j2\hat{\mathbf{y}}$ and $\mathbf{B} = 2\hat{\mathbf{x}} + j\hat{\mathbf{y}}$. Are $\mathcal{A}(t)$ and $\mathcal{B}(t)$ perpendicular at all times?

CHAPTER 1

1.1 (a) $\underline{a}+\underline{b}=5+j3$ (b) $\underline{a}-\underline{b}=11+j$ (c) $\underline{a}\cdot\underline{b}=-26+j2$ (d) $\underline{a}/\underline{b}=-2.2-j1.4$

1.2 $\underline{a}=8+j2=8.25\angle 14.04^\circ$, $\underline{b}=-3+j=3.16\angle 161.57^\circ$

$\underline{a}\cdot\underline{b}=26.08\angle 175.61^\circ$, $\underline{a}/\underline{b}=2.61\angle -147.53^\circ$

1.3 $Re\{e^{j\omega t}\}=\cos\omega t$, $Im\{e^{j\omega t}\}=\sin\omega t$, $|e^{j\omega t}|=1$

1.4 (a) yes (b) yes (c) yes

1.5 $Z^2=1+j=\sqrt{2}\,e^{j(2n\pi+\pi/4)} \Rightarrow Z=2^{1/4}e^{j(n\pi+\pi/8)}$, $n=0,1$

$\therefore Z_1=2^{1/4}e^{j\pi/8}$, $Z_2=2^{1/4}e^{j(\pi+\pi/8)}=-2^{1/4}e^{j\pi/8}$

1.6 $(1+ja)^{1/2}=\pm\{1+\frac{1}{2}(ja)+\frac{1}{8}a^2+\cdots\}\approx\pm\{1+j(a/2)\}$, if $|a|\ll 1$

1.7 $a\gg 1$, $(1+ja)^{1/2}\approx(ja)^{1/2}=\pm\sqrt{j}\,(a)^{1/2}=\pm(1+j)(a/2)^{1/2}$

1.8 (a) $\underline{V}=6e^{j\pi/4}$ (b) $\underline{I}=j8$ (c) $\underline{A}=-j3-2$ (d) $\underline{C}=-j6$ (e) impossible (f) impossible

1.9 (a) $C(t)=Re\{(1+j)e^{j\omega t}\}=Re\{\sqrt{2}\,e^{j\pi/4}e^{j\omega t}\}=\sqrt{2}\cos(\omega t+\pi/4)$

(b) $C(t)=Re\{4e^{j0.8}e^{j\omega t}\}=4\cos(\omega t+0.8)$

(c) $C(t)=Re\{3e^{j\pi/2}e^{j\omega t}+4e^{j0.8}e^{j\omega t}\}=3\cos(\omega t+\pi/2)+4\cos(\omega t+0.8)$

1.10 $\underline{V}=r+jx \Rightarrow V(t)=r\cos\omega t-x\sin\omega t$; $\underline{U}=g+jy \Rightarrow U(t)=g\cos\omega t-y\sin\omega t$

$Re\{\underline{V}\,\underline{U}e^{j\omega t}\}=Re\{[(rg-xy)+j(gx+ry)]e^{j\omega t}\}=(rg-xy)\cos\omega t-(gx+ry)\sin\omega t$

But $V(t)U(t)=rg\cos^2\omega t+xy\sin^2\omega t-(gx+ry)\sin\omega t\cos\omega t\neq Re\{\underline{V}\,\underline{U}e^{j\omega t}\}$

1.11 (a) $\bar{A}+\bar{B}=-6\hat{x}+5\hat{y}+2\hat{z}$ (b) $\bar{A}-\bar{B}=-10\hat{x}+13\hat{y}-4\hat{z}$ (c) $\bar{A}\cdot\bar{B}=-16-36-3=-55$

(d) $\bar{A}\times\bar{B}=23\hat{x}+22\hat{y}+14\hat{z}$

1.12 $|\bar{A}|=|-8\hat{x}+9\hat{y}-\hat{z}|=\sqrt{146}$, $|\bar{B}|=|2\hat{x}-4\hat{y}+3\hat{z}|=\sqrt{29}$

$\cos\alpha=\dfrac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|}=\dfrac{-55}{\sqrt{146}\times\sqrt{29}}=-0.845$ $\therefore\alpha=\cos^{-1}(-0.845)=147.7^\circ$

1.13 $\frac{\partial}{\partial t}V(t)=\frac{\partial}{\partial t}[V_o\cos(\omega t+\phi)]=-V_o\omega\sin(\omega t+\phi)=Re\{jV_o\omega e^{j\phi}e^{j\omega t}\}\longleftrightarrow jV_o\omega e^{j\phi}$

But $V(t)=V_o\cos(\omega t+\phi)=Re\{V_oe^{j\phi}e^{j\omega t}\}=Re\{\underline{V}e^{j\omega t}\}\Rightarrow\underline{V}=V_oe^{j\phi}$

$\therefore\frac{\partial}{\partial t}V(t)\longleftrightarrow j\omega V_oe^{j\phi}=j\omega\underline{V}$

1.14 Let $\bar{C}=x\hat{x}+y\hat{y}$ $\quad\bar{C}\perp\bar{A}\Rightarrow\bar{C}\cdot\bar{A}=0$, where $\bar{A}=-8\hat{x}+9\hat{y}-\hat{z}$

$\bar{C}\cdot\bar{A}=-8x+9y=0$ $\quad\therefore x=\frac{9}{8}y$ $\quad$ Also $|\bar{A}|=1\Rightarrow x^2+y^2=1\Rightarrow y^2=\frac{64}{185}\Rightarrow y=\pm\frac{8}{\sqrt{145}}$

$\therefore x=\pm\frac{9}{\sqrt{145}}$ and $\bar{C}=\pm\frac{1}{\sqrt{165}}(9\hat{x}+8\hat{y})$

1.15 Let $\bar{C}=x\hat{x}+y\hat{y}+z\hat{z}$ $\quad\bar{C}\parallel\bar{A}\Rightarrow\bar{A}\times\bar{C}=0$, where $\bar{A}=5\hat{x}-8\hat{y}+2\hat{z}$

$\bar{A}\times\bar{C}=(-8z-2y)\hat{x}+(2x-5z)\hat{y}+(5y+8x)\hat{z}=0$

$$\therefore\left.\begin{array}{l}-2y-8z=0\\ 2x-5z=0\\ 5y+8x=0\end{array}\right\}\Rightarrow x=\frac{5}{2}z \text{ and } y=-4z$$

But $|\bar{C}|=1\Rightarrow x^2+y^2+z^2=1\Rightarrow z^2(26/4+16+)=1$ $\quad\therefore z=\pm\frac{2}{\sqrt{93}}$

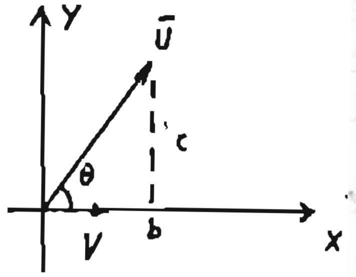
$\therefore\ \bar{C}=\pm\frac{1}{\sqrt{93}}(5\hat{x}-8\hat{y}+2\hat{z})$

1.16 $\overline{AB} = (-1-1)\hat{x} + (3-0)\hat{y} + (-2-2)\hat{z}$

$= -2\hat{x} + 3\hat{y} - 4\hat{z}$

$\hat{n} = (-2\hat{x} + 3\hat{y} - 4\hat{z})/\sqrt{4+9+16} = \frac{1}{\sqrt{29}}(-2\hat{x} + 3\hat{y} - 4\hat{z})$

1.17 (1.10a) gives $\bar{V}\cdot\bar{U} = ab$



(1.10b) gives $\bar{V}\cdot\bar{U} = a\sqrt{b^2+c^2}\cos\theta$

but $\cos\theta = b/\sqrt{b^2+c^2}$

$\therefore \bar{V}\cdot\bar{U} = a\sqrt{b^2+c^2}\cos\theta = ab$

1.18 Let $\bar{A} = A_1\hat{x} + A_2\hat{y} + A_3\hat{z}$

$\bar{B} = B_1\hat{x} + B_2\hat{y} + B_3\hat{z}$

$\bar{C} = C_1\hat{x} + C_2\hat{y} + C_3\hat{z}$

$\bar{A}\cdot(\bar{B}+\bar{C}) = \bar{A}\cdot[(B_1+C_1)\hat{x} + (B_2+C_2)\hat{y} + (B_3+C_3)\hat{z}]$

$= A_1B_1 + C_1A_1 + A_2B_2 + A_2C_2 + A_3B_3 + A_3C_3$

$\bar{A}\cdot\bar{B} + \bar{A}\cdot\bar{C} = A_1B_1 + A_2B_2 + A_3B_3 + A_1C_1 + A_2C_2 + A_3C_3$

Therefore $\bar{A}\cdot(\bar{B}+\bar{C}) = \bar{A}\cdot\bar{B} + \bar{A}\cdot\bar{C}$

1.19 (1.12) gives $\bar{V}\times\bar{U} = ac\hat{z}$

(1.14) gives $\bar{V}\times\bar{U} = a\sqrt{b^2+c^2}\sin\theta\,\hat{z}$

but $\sin\theta = c/\sqrt{b^2+c^2}$

therefore $\bar{V}\times\bar{U} = a\sqrt{b^2+c^2}\sin\theta\,\hat{z} = ac\hat{z}$

1.20 $\bar{V}\times\bar{U} = vu\sin(135°)\hat{z} = \frac{\sqrt{2}}{2}vu\hat{z}$

$V\times\bar{U} = vu\sin(225°)(-\hat{z}) = \frac{\sqrt{2}}{2}vu\hat{z}$

1.21 Let $\bar{A} = A_1\hat{x} + A_2\hat{y} + A_3\hat{z}$, $\bar{B} = B_1\hat{x} + B_2\hat{y} + B_3\hat{z}$, and $\bar{C} = C_1\hat{x} + C_2\hat{y} + C_3\hat{z}$, then

$$\bar{B}\times\bar{C} = \hat{x}(B_2C_3 - C_2B_3) + \hat{y}(B_3C_1 - C_3B_1) + \hat{z}(B_1C_2 - C_1B_2)$$

$$\begin{aligned}\bar{A}\times(\bar{B}\times\bar{C}) = {} & \hat{x}[A_2(B_1C_2 - C_1B_2) - A_3(B_3C_1 - C_3B_1)] \\ & + \hat{y}[A_3(B_2C_3 - C_2B_3) - A_1(B_1C_2 - B_2C_1)] \\ & + \hat{z}[A_1(B_3C_1 - B_1C_3) - A_2(B_2C_3 - B_3C_2)]\end{aligned}$$

On the other hand,

$$\bar{B}(\bar{A}\cdot\bar{C}) = (B_1\hat{x} + B_2\hat{y} + B_3\hat{z})(A_1C_1 + A_2C_2 + A_3C_3)$$

$$\bar{C}(\bar{A}\cdot\bar{B}) = (C_1\hat{x} + C_2\hat{y} + C_3\hat{z})(A_1B_1 + A_2B_2 + A_3B_3)$$

$$\therefore\ \bar{B}(\bar{A}\cdot\bar{C}) - \bar{C}(\bar{A}\cdot\bar{B})$$

$$\begin{aligned} = {} & \hat{x}[B_1(\cancel{A_1C_1} + A_2C_2 + A_3C_3) - C_1(\cancel{A_1B_1} + A_2B_2 + A_3B_3)] \\ & + \hat{y}[B_2(A_1C_1 + \cancel{A_2C_2} + A_3C_3) - C_2(A_1B_1 + \cancel{A_2B_2} + A_3B_3)] \\ & + \hat{z}[B_3(A_1C_1 + A_2C_2 + \cancel{A_3C_3}) - C_3(A_1B_1 + A_2B_2 + \cancel{A_3B_3})]\end{aligned}$$

Therefore $\bar{A}\times(\bar{B}\times\bar{C}) = \bar{B}(\bar{A}\cdot\bar{C}) - \bar{C}(\bar{A}\cdot\bar{B})$

1.22

(a) $\underline{\bar{V}} = 3\hat{x} - j4\hat{y} + j\hat{z}$

(b) $\underline{\bar{E}} = (3-j4)\hat{x} + 8(1+j)\hat{z}$

(c) $\underline{\bar{H}} = 0.5\,e^{-j4z}\,\hat{x}$

1.23 $\frac{\partial}{\partial t}\bar{E}(z,t) \leftrightarrow j\varepsilon\bar{E} = j\omega[(3-j4)\hat{x} + 8(1+j)\hat{z}]$

1.24 (a) $\bar{C}(t) = \hat{x}\cos\omega t + \hat{y}\sin\omega t$

(b) $\bar{C}(t) = -\hat{x}\sin\omega t + \hat{y}\cos\omega t$

(c) $\bar{C}(t) = \hat{x}\cos(\omega t - kz) - \hat{y}\sin(\omega t + kz)$

1.25 (a) $\underline{A} + \underline{B} = (-1+j3)\hat{y} + (1+j3)\hat{z}$

(b) $\underline{A} - \underline{B} = 2\hat{x} + (1-j)\hat{y} + (1+j)\hat{z}$

(c) $\underline{A}\cdot\underline{B} = -1-(j+2)+(j-2) = -5$

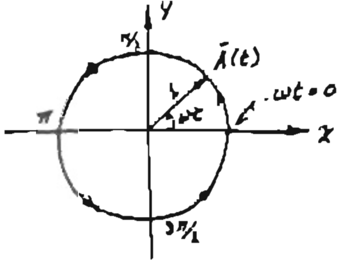
(d) $\underline{A}\times\underline{B} = 4\hat{x} - (1+j3)\hat{y} - (1-j3)\hat{z}$

1.26 $\underline{A}\cdot\underline{A}^* = [\hat{x}+j\hat{y}+(1+j2)\hat{z}]\cdot[\hat{x}-j\hat{y}+(1-j2)\hat{z}] = 1+1+5 = 7$

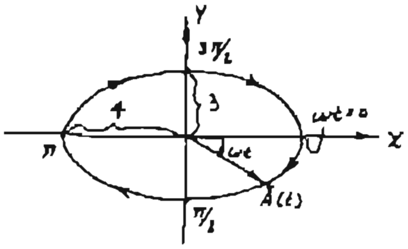
$\underline{A}\times\underline{B}^* = [\hat{x}+j\hat{y}+(1+j2)\hat{z}]\times[-\hat{x}-(1+j2)\hat{y}-j\hat{z}] = (-2+j4)\hat{x} - (1+j)\hat{y} - (1+j)\hat{z}$

$\therefore Re\{\underline{A}\times\underline{B}^*\} = -2\hat{x} - \hat{y} - \hat{z}$

1.27 (a) $\underline{A} = \hat{x} - j\hat{y} \longleftrightarrow \bar{A}(t) = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$ and $|\bar{A}(t)| = 1$



(b) $\underline{A} = 4\hat{x} + j3\hat{y} \longleftrightarrow \bar{A}(t) = 4\cos\omega t\,\hat{x} - 3\sin\omega t\,\hat{y}$



1.28 $\underline{A} = \hat{x} + j2\hat{y}$, $\underline{B} = 2\hat{x} + j\hat{y} \rightarrow \underline{A}\cdot\underline{B} = 2-2 = 0$

But $\bar{A}(t) = \cos\omega t\,\hat{x} - 2\sin\omega t\,\hat{y}$ and $\bar{B}(t) = 2\cos\omega t\,\hat{x} - \sin\omega t\,\hat{y} \Rightarrow \bar{A}(t)\cdot\bar{B}(t) = 2$

$\therefore$ $\bar{A}(t)$ and $\bar{B}(t)$ are not perpendicular at any time.

Solution:

$$E(t) = \mathrm{Re}\{\hat{x}E_0 e^{-jkz} e^{j\omega t}\} = \hat{x}E_0 \cos(\omega t - kz)$$

$$H(t) = \mathrm{Re}\{\hat{y}\frac{k}{\omega\mu} E_0 e^{-jkz} e^{j\omega t}\} = \hat{y}\frac{k}{\omega\mu} E_0 \cos(\omega t - kz)$$

? $$S(t) = E \times H = \hat{z}\frac{k}{\omega\mu} E_0^2 \cos^2(\omega t - kz)$$

$$\langle S\rangle = \frac{1}{2}\mathrm{Re}\{E \times H^*\} = \hat{z}\frac{k}{2\omega\mu} E_0^2$$

$$U_E = \frac{\epsilon E_0^2}{2}\cos^2(\omega t - kz)$$

$$U_H = \frac{k^2E_0^2}{2\omega^2\mu}\cos^2(\omega t - kz)$$

$$\langle U_E\rangle = \frac{\epsilon}{4}E_0^2$$

$$\langle U_H\rangle = \frac{k^2}{4\omega^2\mu}E_0^2$$

Problems

2.1 Let $A = 5\hat{x} + 6yz\hat{y} + x^2\hat{z}$; find $\nabla \times A$ and $\nabla \cdot A$.

2.2 Let $\phi = xyz$; find $\nabla\phi$ and $\nabla \cdot \nabla\phi$.

2.3 Let $a = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}$ and $b = b_1\hat{x} + b_2\hat{y} + b_3\hat{z}$. Show that equations (2.9), (2.11a), and (2.12) are true.

2.4 Show that $\nabla \times (a + b) = \nabla \times a + \nabla \times b$ and $\nabla \cdot (a + b) = \nabla \cdot a + \nabla \cdot b$.

2.5 Show that $\nabla(\Phi_1 \Phi_2) = \Phi_1 \nabla \Phi_2 + \Phi_2 \nabla \Phi_1$ and that $\nabla \cdot (\Phi A) = A \cdot \nabla\Phi + \Phi\nabla \cdot A$.

2.6 Show that $\nabla \times (\Phi A) = \nabla\Phi \times A + \Phi\nabla \times A$.

2.7 In a source-free region, $H = z\hat{y} + y\hat{z}$. Does D vary with time?

2.8 What is the charge density in a region where $D = 2x\hat{x}$?

2.9 Find the magnetic field $B(y,t)$ associated with the electric field $E(y,t)$ given as follows:

$$E(y,t) = \hat{x}\,0.3\cos(\omega t + ky)$$

where ω and k are constants.

2.10 Express k in terms of the magnetic permeability and dielectric permittivity of the medium when the electromagnetic fields are given in Problem 2.9 in a source-free region.

2.11 Let E_1, B_1, H_1, and D_1 satisfy equations (2.1)–(2.4) with given J_1 and ρ_{v1}. Let also E_2, B_2, H_2, and D_2 satisfy equations (2.1)–(2.4) with given J_2 and ρ_{v2}. What are the electromagnetic fields due to a current J_t and charge ρ_{vt} where $J_t = J_1 + J_2$ and $\rho_{vt} = \rho_{v1} + \rho_{v2}$? You must show that your proposed solution satisfies Maxwell's equations. What is the appropriate name for the theorem you have just proved?

2.12 (a) It is known that the vector $\mathbf{a}$ is equal to zero at one point. Does that imply that $\nabla \times \mathbf{a} = 0$ at that point? Give a counter-example if your answer is no.

(b) Does $\mathbf{E} = 0$ on a line always imply $\nabla \times \mathbf{E} = 0$ on that line? Give a counter-example if the answer is no.

(c) It is found that the $\mathbf{E}$ field is zero on a surface. Does it follow that $\partial \mathbf{B}/\partial t = 0$ on that surface?

2.13 Show that equations (2.22c) and (2.22d) can be derived from equations (2.22a), (2.22b), and the conservation equation (2.23).

2.14 To represent time-harmonic fields, most physics books use the factor $e^{-i\omega t}$ instead of $e^{j\omega t}$, which most electrical engineering books use. For a time-harmonic real function $A(x, y, z, t) = a(x, y, z) \cos(\omega t + \phi)$, find the phasor notation that corresponds to the physicists' convention. What is the corresponding conversion rule by which phasors can be transformed back to the real-time expression?

2.15 Refer to Problem 2.14 about the notation $e^{-i\omega t}$ adopted in most physics books. Write the time-harmonic Maxwell's equations using that notation.

2.16 What is the range of effective permittivity of the ionosphere at AM broadcasting frequencies? Use the following data: $N = 10^{12}\ \mathrm{m}^{-3}$ and $f = 500$ kHz to 1 MHz.

2.17 Show that the dimension of each term of equation (2.36) is watts per cubic meter.

2.18 Indicate in watts, meters, and joules the dimensions of the following quantities: (a) $\mathbf{E} \cdot \mathbf{D}$, (b) $\mathbf{H} \cdot \mathbf{B}$, and (c) $\mathbf{S}$.

2.19 Let $\mathbf{E} = (\hat{x} + j\hat{y})\, e^{-jz}$ and $\mathbf{H} = (\hat{y} - j\hat{x})\, e^{-jz}$. Find $\mathbf{S}$ in terms of z and ωt and find $\langle \mathbf{S} \rangle$.

2.20 Show that $\mathbf{S} \neq \mathrm{Re}\,\{\mathbf{E} \times \mathbf{H}\, e^{j\omega t}\}$.

2.21 Show that $\mathbf{S} \neq \mathrm{Re}\,\{\mathbf{E}\, e^{j\omega t} \times \mathbf{H}\, e^{j\omega t}\}$.

2.22 Compare the energy stored in a cubic region one meter on a side which has a uniform E field of 10^4 V/m to the energy stored in a similar region with a uniform B field of 10^4 G. (One G $= 10^{-4}\ \mathrm{Wb/m^2}$). The medium is air.

2.23 Repeat Problem 2.22 for the case where the medium is water instead of air. Use $\epsilon = 80\,\epsilon_0$ and $\mu = \mu_0$ for water.

CHAPTER 2

2.1 $\nabla\times\bar{A} = -6y\,\hat{x} - 3x^2\,\hat{y}\,, \qquad \nabla\cdot\bar{A} = 6z$

2.2 $\nabla\phi = yz\,\hat{x} + xz\,\hat{y} + xy\,\hat{z}\,, \qquad \nabla\cdot\nabla\phi = 0$

2.3 $\bar{a} = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}\,, \quad \bar{b} = b_1\hat{x} + b_2\hat{y} + b_3\hat{z}$

$$\nabla\times\bar{a} = \hat{x}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + \hat{y}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + \hat{z}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$\nabla\times(\nabla\times\bar{a}) = \hat{x}\left(-\frac{\partial^2 a_1}{\partial y^2} + \frac{\partial^2 a_2}{\partial y\partial x} - \frac{\partial^2 a_1}{\partial z^2} + \frac{\partial^2 a_3}{\partial z\partial x}\right) + \hat{y}\left(\frac{\partial^2 a_3}{\partial y\partial z} - \frac{\partial^2 a_2}{\partial z^2} + \frac{\partial^2 a_1}{\partial x\partial y} - \frac{\partial^2 a_2}{\partial x^2}\right)$$
$$+ \hat{z}\left(-\frac{\partial^2 a_3}{\partial y^2} + \frac{\partial^2 a_2}{\partial y\partial z} + \frac{\partial^2 a_1}{\partial x\partial z} - \frac{\partial^2 a_3}{\partial x^2}\right)$$

$$\nabla\cdot\bar{a} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}\,, \quad \nabla(\nabla\cdot\bar{a}) = \hat{x}\left(\frac{\partial^2 a_1}{\partial x^2} + \frac{\partial^2 a_2}{\partial x\partial y} + \frac{\partial^2 a_3}{\partial x\partial z}\right) + \hat{y}\left(\frac{\partial^2 a_1}{\partial x\partial y} + \frac{\partial^2 a_2}{\partial y^2} + \frac{\partial^2 a_3}{\partial y\partial z}\right) + \hat{z}\left(\frac{\partial^2 a_1}{\partial x\partial z} + \frac{\partial^2 a_1}{\partial y\partial z} + \frac{\partial^2 a_3}{\partial z^2}\right)$$

$$\nabla^2\bar{a} = \hat{x}\left(\frac{\partial^2 a_1}{\partial x^2} + \frac{\partial^2 a_1}{\partial y^2} + \frac{\partial^2 a_1}{\partial z^2}\right) + \hat{y}\left(\frac{\partial^2 a_2}{\partial x^2} + \frac{\partial^2 a_2}{\partial y^2} + \frac{\partial^2 a_2}{\partial z^2}\right) + \hat{z}\left(\frac{\partial^2 a_3}{\partial x^2} + \frac{\partial^2 a_3}{\partial y^2} + \frac{\partial^2 a_3}{\partial z^2}\right)$$

$$\therefore\ \nabla\times(\nabla\times\bar{a}) = \nabla(\nabla\cdot\bar{a}) - \nabla^2\bar{a}$$

$$\nabla\cdot(\nabla\times\bar{a}) = \cancel{\frac{\partial^2 a_3}{\partial x\partial y}} - \frac{\partial^2 a_1}{\partial x\partial z} + \frac{\partial^2 a_1}{\partial y\partial z} - \cancel{\frac{\partial^2 a_3}{\partial x\partial y}} + \frac{\partial^2 a_2}{\partial x\partial z} - \frac{\partial^2 a_1}{\partial y\partial z} = 0$$

$$\bar{a}\times\bar{b} = \hat{x}(a_2b_3 - a_3b_2) + \hat{y}(a_3b_1 - a_1b_3) + \hat{z}(a_1b_2 - a_2b_1)$$

$$\nabla\cdot(\bar{a}\times\bar{b}) = \frac{\partial}{\partial x}(a_2b_3 - a_3b_2) + \frac{\partial}{\partial y}(a_3b_1 - a_1b_3) + \frac{\partial}{\partial z}(a_1b_2 - a_2b_1) = a_2\left(\frac{\partial b_3}{\partial x} - \frac{\partial b_1}{\partial z}\right) + b_3\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$
$$- a_3\left(\frac{\partial b_1}{\partial y} - \frac{\partial b_2}{\partial x}\right) + b_2\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + b_1\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + a_1\left(\frac{\partial b_2}{\partial z} - \frac{\partial b_3}{\partial y}\right)$$

$$\bar{b}\cdot(\nabla\times\bar{a}) = b_1\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + b_2\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + b_3\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$\bar{a}\cdot(\nabla\times\bar{b}) = a_1\left(\frac{\partial b_3}{\partial y} - \frac{\partial b_2}{\partial z}\right) + a_2\left(\frac{\partial b_1}{\partial z} - \frac{\partial b_3}{\partial x}\right) + a_3\left(\frac{\partial b_2}{\partial x} - \frac{\partial b_1}{\partial y}\right)$$

$$\therefore\ \nabla\cdot(\bar{a}\times\bar{b}) = \bar{b}\cdot\nabla\times\bar{a} - \bar{a}\cdot\nabla\times\bar{b}$$

2.4 Let $\bar{a} = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}\,, \quad \bar{b} = b_1\hat{x} + b_2\hat{y} + b_3\hat{z}$

$$\nabla\times(\bar{a}+\bar{b}) = \nabla\times\left[(a_1+b_1)\hat{x} + (a_2+b_2)\hat{y} + (a_3+b_3)\hat{z}\right] = \hat{x}\left[\frac{\partial}{\partial y}(a_3+b_3) - \frac{\partial}{\partial z}(a_2+b_2)\right]$$
$$+ \hat{y}\left[\frac{\partial}{\partial z}(a_1+b_1) - \frac{\partial}{\partial x}(a_3+b_3)\right] + \hat{z}\left[\frac{\partial}{\partial x}(a_2+b_2) - \frac{\partial}{\partial y}(a_1+b_1)\right]$$

$$\nabla\times\bar{a} = \hat{x}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + \hat{y}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + \hat{z}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$\nabla\times\bar{b} = \hat{x}\left(\frac{\partial b_3}{\partial y} - \frac{\partial b_2}{\partial z}\right) + \hat{y}\left(\frac{\partial b_1}{\partial z} - \frac{\partial b_3}{\partial x}\right) + \hat{z}\left(\frac{\partial b_2}{\partial x} - \frac{\partial b_1}{\partial y}\right)$$

$$\therefore\ \nabla\times(\bar{a}+\bar{b}) = \nabla\times\bar{a} + \nabla\times\bar{b}$$

$2 \neq 0$

$$\nabla\cdot(\bar{a}+\bar{b}) = \frac{\partial}{\partial x}(a_1+b_1) + \frac{\partial}{\partial y}(a_2+b_2) + \frac{\partial}{\partial z}(a_3+b_3) = \left(\frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}\right) + \left(\frac{\partial b_1}{\partial x} + \frac{\partial b_2}{\partial y} + \frac{\partial b_3}{\partial z}\right) = \nabla\cdot\bar{a} + \nabla\cdot\bar{b}$$

2.5 $$\nabla(\phi_1\phi_2) = \hat{x}\frac{\partial}{\partial x}(\phi_1\phi_2) + \hat{y}\frac{\partial}{\partial y}(\phi_1\phi_2) + \hat{z}\frac{\partial}{\partial z}(\phi_1\phi_2) = \hat{x}\left[\phi_1\frac{\partial\phi_2}{\partial x} + \phi_2\frac{\partial\phi_1}{\partial x}\right] + \hat{y}\left[\phi_1\frac{\partial\phi_2}{\partial y} + \phi_2\frac{\partial\phi_1}{\partial y}\right]$$
$$+ \hat{z}\left[\phi_1\frac{\partial\phi_2}{\partial z} + \phi_2\frac{\partial\phi_1}{\partial z}\right] = \phi_1\left[\hat{x}\frac{\partial\phi_2}{\partial x} + \hat{y}\frac{\partial\phi_2}{\partial y} + \hat{z}\frac{\partial\phi_2}{\partial z}\right] + \phi_2\left[\hat{x}\frac{\partial\phi_1}{\partial x} + \hat{y}\frac{\partial\phi_1}{\partial y} + \hat{z}\frac{\partial\phi_1}{\partial z}\right] = \phi_1\nabla\phi_2 + \phi_2\nabla\phi_1$$

Let $\bar{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$

$$\nabla\cdot(\phi\bar{A}) = \frac{\partial}{\partial x}(\phi A_x) + \frac{\partial}{\partial y}(\phi A_y) + \frac{\partial}{\partial z}(\phi A_z) = \phi\left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right] + \left[\frac{\partial\phi}{\partial x}A_x + \frac{\partial\phi}{\partial y}A_y + \frac{\partial\phi}{\partial z}A_z\right]$$
$$= \phi\nabla\cdot\bar{A} + \nabla\phi\cdot\bar{A}$$

2.6 $$\nabla\times(\phi\bar{A}) = \hat{x}\left[\frac{\partial}{\partial y}(\phi A_z) - \frac{\partial}{\partial z}(\phi A_y)\right] + \hat{y}\left[\frac{\partial}{\partial z}(\phi A_x) - \frac{\partial}{\partial x}(\phi A_z)\right] + \hat{z}\left[\frac{\partial}{\partial x}(\phi A_y) - \frac{\partial}{\partial y}(\phi A_x)\right]$$
$$= \phi\left[\hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\right] + \hat{x}\left(A_z\frac{\partial\phi}{\partial y} - A_y\frac{\partial\phi}{\partial z}\right)$$
$$+ \hat{y}\left(A_x\frac{\partial\phi}{\partial z} - A_z\frac{\partial\phi}{\partial x}\right) + \hat{z}\left(A_y\frac{\partial\phi}{\partial x} - A_x\frac{\partial\phi}{\partial y}\right)$$
$$= \phi\nabla\times\bar{A} + \nabla\phi\times\bar{A}$$

2.7 $\nabla\times\bar{H} = \nabla\times(z\hat{y} + y\hat{z}) = \hat{x}(1-1) = 0 \Rightarrow \dfrac{\partial\bar{D}}{\partial t} = 0$

$\therefore\ \bar{D}$ is not varying with time.

2.8 $\nabla\cdot\bar{D} = \nabla\cdot(2x\hat{x}) = 2 \Rightarrow \rho_v = 2\ C/m^3$

2.9 $\bar{E} = 0.3e^{jky}\hat{x}$

$\nabla\times\bar{E} = -\hat{z}(0.3jke^{jky}) = -j\omega\bar{B}$

$\bar{B} = 0.3\frac{k}{\omega}e^{jky}\hat{z}$

$\bar{B}(y,t) = \frac{0.3k}{\omega}\cos(\omega t + ky)\hat{z}$

2.10 $\nabla\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}$, substituting $\bar{H} = \frac{1}{\mu}\bar{B} = \frac{0.3k}{\omega\mu}\cos(\omega t + ky)\hat{z}$, $\bar{J} = 0$, $\bar{D} = \varepsilon\bar{E} = 0.3\varepsilon\cos(\omega t + ky)\hat{x}$ into this equation yields:

$k^2 = \omega^2\mu\varepsilon$ or $k = \omega\sqrt{\mu\varepsilon}$.

2.11 $\nabla\times\bar{E}_1 = -\frac{\partial}{\partial t}\bar{B}_1$, $\nabla\times\bar{H}_1 = \bar{J}_1 + \frac{\partial}{\partial t}\bar{D}_1$, $\nabla\cdot\bar{B}_1 = 0$, $\nabla\cdot\bar{D}_1 = \rho_{v1}$

$\nabla\times\bar{E}_2 = -\frac{\partial}{\partial t}\bar{B}_2$, $\nabla\times\bar{H}_2 = \bar{J}_2 + \frac{\partial}{\partial t}\bar{D}_2$, $\nabla\cdot\bar{B}_2 = 0$, $\nabla\cdot\bar{D}_2 = \rho_{v2}$

$\nabla\times\bar{E}_t = \nabla\times(\bar{E}_1 + \bar{E}_2) = \nabla\times\bar{E}_1 + \nabla\times\bar{E}_2 = -\frac{\partial}{\partial t}\bar{B}_1 - \frac{\partial}{\partial t}\bar{B}_2 = -\frac{\partial}{\partial t}(\bar{B}_1 + \bar{B}_2) = -\frac{\partial}{\partial t}\bar{B}_t$

$\nabla\times\bar{H}_t = \nabla\times(\bar{H}_1 + \bar{H}_2) = \nabla\times\bar{H}_1 + \nabla\times\bar{H}_2 = \bar{J}_1 + \frac{\partial}{\partial t}\bar{D}_1 + \bar{J}_2 + \frac{\partial}{\partial t}\bar{D}_2 = (\bar{J}_1 + \bar{J}_2) + \frac{\partial}{\partial t}(\bar{D}_1 + \bar{D}_2) = \bar{J}_t + \frac{\partial}{\partial t}\bar{D}_t$

$\nabla\cdot\bar{B}_t = \nabla\cdot(\bar{B}_1 + \bar{B}_2) = \nabla\cdot\bar{B}_1 + \nabla\cdot\bar{B}_2 = 0 + 0 = 0$

$\nabla\cdot\bar{D}_t = \nabla\cdot(\bar{D}_1 + \bar{D}_2) = \nabla\cdot\bar{D}_1 + \nabla\cdot\bar{D}_2 = \rho_{v1} + \rho_{v2} = \rho_{vt}$

$\therefore$ $\bar{E}_t = \bar{E}_1 + \bar{E}_2$, $\bar{H}_t = \bar{H}_1 + \bar{H}_2$, $\bar{B}_t = \bar{B}_1 + \bar{B}_2$ and $\bar{D}_t = \bar{D}_1 + \bar{D}_2$ satisfy Maxwell's equations w sources $\bar{J}_t = \bar{J}_1 + \bar{J}_2$ and $\rho_{vt} = \rho_{v1} + \rho_{v2}$.

This is the superposition theorem.

2.12 (a) No. $\bar{a} = \hat{x}\sin z + \hat{y}\sin x + \hat{z}\sin y = 0$ at $(0,0,0)$, but $\nabla\times\bar{a} = \hat{x}\cos y + \hat{y}\cos z + \hat{z}\cos$ at $(0,0,0)$.

(b) No. $\bar{E} = \hat{x}\sin z + \hat{y}\sin x = 0$ on y-axis, but $\nabla\times\bar{E} = \hat{y}\cos z + \hat{z}\cos x \neq 0$ on y-axis

(c) NO. $\bar{E} = \hat{x}\sin z = 0$ on $z=0$ plane, but $\nabla\times\bar{E} = \hat{y}\cos z \neq 0$ on $z=0$ plane $\Rightarrow \frac{\partial\bar{B}}{\partial t} \neq 0$ on that plane

2.13 $\nabla\cdot(\nabla\times\bar{E}) \equiv 0 \Rightarrow j\omega\nabla\cdot\bar{B} = 0 \Rightarrow \nabla\cdot\bar{B} = 0$

$\nabla\cdot(\nabla\times\bar{H}) \equiv 0 \Rightarrow \nabla\cdot\bar{J} + j\omega\nabla\cdot\bar{D} = 0 \Rightarrow j\omega(-\rho_v + \nabla\cdot\bar{D}) = 0 \Rightarrow \nabla\cdot\bar{D} = \rho_v$

2.14 $A(x,y,z,t) = a(x,y,z)\cos(\omega t + \phi) = a(x,y,z)\cos(-\omega t - \phi)$

$e^{j\omega t}$: $A(x,y,z,t) = Re\{a(x,y,z)e^{j\phi}e^{j\omega t}\} \longleftrightarrow a(x,y,z)e^{j\phi}$

$e^{-i\omega t}$: $A(x,y,z,t) = Re\{a(x,y,z)e^{-i\phi}e^{-i\omega t}\} \longleftrightarrow a(x,y,z)e^{-i\phi}$

2.15 $\nabla\times\bar{E} = i\omega\bar{B}$, $\nabla\times\bar{H} = \bar{J} - i\omega\bar{D}$, $\nabla\cdot\bar{B} = 0$, $\nabla\cdot\bar{D} = \rho_v$

2.16 $\varepsilon = \varepsilon_0(1 - \frac{\omega_p^2}{\omega^2})$ $\omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}} = \left[\frac{10^{12}\times(1.6\times10^{-19})^2}{9.11\times10^{-31}}\times36\pi\times10^9\right]^{1/2} = 5.64\times10^7\ Hz$

at 500 kHz: $\varepsilon = \varepsilon_0\left[1 - \left(\frac{5.64\times10^7}{2\pi\times5\times10^6}\right)^2\right] = -323\ \varepsilon_0$

at 1 MHz: $\varepsilon = \varepsilon_0\left[1 - \left(\frac{5.64\times10^7}{2\pi\times10^6}\right)^2\right] = -80.43\ \varepsilon_0$

2.17 $[\bar{E}] = \mathrm{V}/\mathrm{m}$, $[\bar{H}] = \mathrm{A}/\mathrm{m}$. $\therefore [\nabla\cdot(\bar{E}\times\bar{H})] = \frac{\mathrm{V\text{-}A}}{\mathrm{m}^3} = \frac{\mathrm{Watt}}{\mathrm{m}^3}$

$[\mu\bar{H}\cdot\bar{H}] = \frac{\mathrm{V\text{-}sec}}{\mathrm{m}^2}\cdot\frac{\mathrm{A}}{\mathrm{m}} = \frac{\mathrm{Watt\text{-}sec}}{\mathrm{m}^3}$, $\therefore \left[\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\bar{H}\cdot\bar{H}\right)\right] = \frac{\mathrm{Watt}}{\mathrm{m}^3}$

$[\varepsilon\bar{E}\cdot\bar{E}] = \frac{\mathrm{C}}{\mathrm{m}^2}\cdot\frac{\mathrm{V}}{\mathrm{m}} = \frac{\mathrm{Joule}}{\mathrm{m}^3}$, $\therefore \left[\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon\bar{E}\cdot\bar{E}\right)\right] = \frac{\mathrm{Watt}}{\mathrm{m}^3}$

$[\bar{J}\cdot\bar{E}] = \frac{\mathrm{A}}{\mathrm{m}^2}\cdot\frac{\mathrm{V}}{\mathrm{m}} = \frac{\mathrm{Watt}}{\mathrm{m}^3}$

2.18 (a) $[\bar{E}\cdot\bar{D}] = \frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{C}}{\mathrm{m}^2} = \frac{\mathrm{Joule}}{\mathrm{m}^3}$

(b) $[\bar{H}\cdot\bar{B}] = \frac{\mathrm{A}}{\mathrm{m}}\frac{\mathrm{V\text{-}sec}}{\mathrm{m}^2} = \frac{\mathrm{Watt\text{-}sec}}{\mathrm{m}^3} = \frac{\mathrm{Joule}}{\mathrm{m}^3}$

(c) $[\bar{S}] = [\bar{E}\times\bar{H}] = \frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{A}}{\mathrm{m}} = \frac{\mathrm{Watt}}{\mathrm{m}^2}$

2.19 $\underline{E} = (\hat{x}+j\hat{y})e^{-jz} \longleftrightarrow \bar{E}(t) = \hat{x}\cos(\omega t - z) - \hat{y}\sin(\omega t - z)$

$\underline{H} = (\hat{y}-j\hat{x})e^{-jz} \longleftrightarrow \bar{H}(t) = \hat{y}\cos(\omega t - z) + \hat{x}\sin(\omega t - z)$

$\bar{S} = \bar{E}\times\bar{H} = \hat{z}\left[\cos^2(\omega t - z) + \sin^2(\omega t - z)\right] = \hat{z}$

$\underline{S} = \underline{E}\times\underline{H}^* = (\hat{x}+j\hat{y})\times(\hat{y}+j\hat{x}) = \hat{z}\,2 \quad \rightarrow \quad \langle\bar{S}\rangle = \frac{1}{2}\mathrm{Re}\{\underline{S}\} = \hat{z}$

2.20 Let $\underline{E} = \bar{E}_R + j\bar{E}_I$ and $\underline{H} = \bar{H}_R + j\bar{H}_I$

$\bar{E} = \mathrm{Re}\{\underline{E}e^{j\omega t}\} = \bar{E}_R\cos\omega t - \bar{E}_I\sin\omega t$, $\bar{H} = \mathrm{Re}\{\underline{H}e^{j\omega t}\} = \bar{H}_R\cos\omega t - \bar{H}_I\sin\omega t$

$\bar{S} = \bar{E}\times\bar{H} = (\bar{E}_R\times\bar{H}_R)\cos^2\omega t + (\bar{E}_I\times\bar{H}_I)\sin^2\omega t - (\bar{E}_R\times\bar{H}_I + \bar{E}_I\times\bar{H}_R)\sin\omega t\cos\omega t$

But $\underline{H}e^{j\omega t} = (\bar{H}_R\cos\omega t - \bar{H}_I\sin\omega t) + j(\bar{H}_R\sin\omega t + \bar{H}_I\cos\omega t)$ and

$\mathrm{Re}\{\underline{E}\times\underline{H}e^{j\omega t}\} = \bar{E}_R\times\bar{H}_R\cos\omega t - \bar{E}_R\times\bar{H}_I\sin\omega t - \bar{E}_I\times\bar{H}_R\sin\omega t - \bar{E}_I\times\bar{H}_I\cos\omega t \neq \bar{S}$

2.21 Let $\underline{E} = \bar{E}_R + j\bar{E}_I$ and $\underline{H} = \bar{H}_R + j\bar{H}_I$

$\underline{E}e^{j\omega t} = (\bar{E}_R\cos\omega t - \bar{E}_I\sin\omega t) + j(\bar{E}_R\sin\omega t + \bar{E}_I\cos\omega t)$

$\underline{H}e^{j\omega t} = (\bar{H}_R\cos\omega t - \bar{H}_I\sin\omega t) + j(\bar{H}_R\sin\omega t + \bar{H}_I\cos\omega t)$

$$\begin{aligned}\mathrm{Re}\{\underline{E}e^{j\omega t}\times\underline{H}e^{j\omega t}\} &= (\bar{E}_R\times\bar{H}_R)\cos^2\omega t + (\bar{E}_I\times\bar{H}_I)\sin^2\omega t - (\bar{E}_R\times\bar{H}_I + \bar{E}_I\times\bar{H}_R)\sin\omega t\cos\omega t\\ &\quad - (\bar{E}_I\times\bar{H}_I)\cos^2\omega t - (\bar{E}_R\times\bar{H}_R)\sin^2\omega t - (\bar{E}_R\times\bar{H}_I + \bar{E}_I\times\bar{H}_R)\sin\omega t\cos\omega t\\ &= (\bar{E}_R\times\bar{H}_R - \bar{E}_I\times\bar{H}_I)(\cos^2\omega t - \sin^2\omega t) - 2(\bar{E}_R\times\bar{H}_I + \bar{E}_I\times\bar{H}_R)\sin\omega t\cos\omega t\end{aligned}$$

$\therefore \bar{S} \neq \mathrm{Re}\{\underline{E}e^{j\omega t}\times\underline{H}e^{j\omega t}\}$

2.22 $U_e = \frac{1}{2}\varepsilon_0\bar{E}\cdot\bar{E} = \frac{1}{2}\times\frac{1}{36\pi}\times10^{-9}\times(10^4)^2 = 4.42\times10^{-4}\,(\mathrm{Joule}/\mathrm{m}^3)$

$U_h = \frac{1}{2}\mu_0\bar{H}\cdot\bar{H} = \frac{1}{2}\frac{1}{\mu_0}\bar{B}\cdot\bar{B} = \frac{1}{2\times4\pi\times10^{-7}}(10^4\times10^{-4})^2 = 3.98\times10^5\,(\mathrm{Joule}/\mathrm{m}^3)$

$\therefore U_h/U_e \doteq 9\times10^8$

2.23 $U_e = \frac{1}{2}\varepsilon\bar{E}\cdot\bar{E} = \frac{1}{2}\times\frac{80}{36\pi}\times10^{-9}\times(10^4)^2 = 3.54\times10^{-2}\,(\mathrm{Joule}/\mathrm{m}^3)$

$U_h = 3.98\times10^5\,(\mathrm{Joule}/\mathrm{m}^3)$

$\therefore U_h/U_e \doteq 1.13\times10^7$

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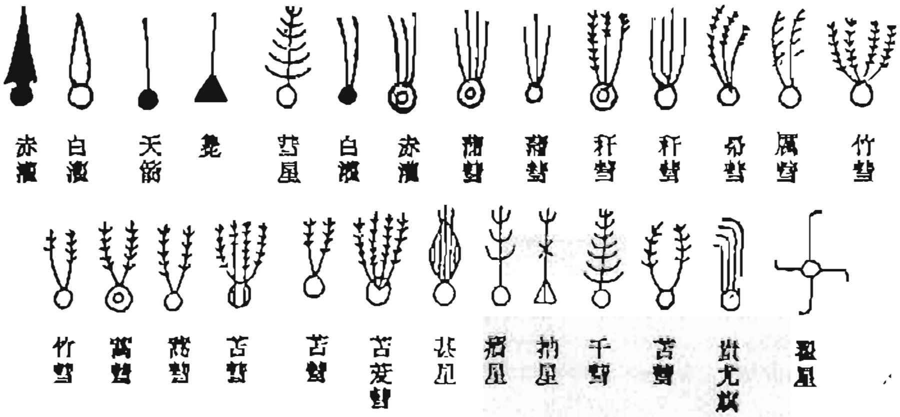


Figure 3.11 Various comet shapes drawn on silk found in China. These figures were painted between 246 to 177 B.C. Below these figures are Chinese names for these comets.

Problems

3.1 Estimate the power density of electromagnetic radiation from the sun received on earth in the same frequency band as that of the VHF television channel 2 (54–60 MHz).

3.2 Consider the sun as an isotropic radiation source. Calculate the total power radiated by the sun in the television channel-2 frequency band (see Problem 3.1). The distance between the sun and the earth is approximately 1.5×10^8 km.

3.3 Assume that solar radiation is isotropic. Estimate the total power radiated by the sun. The solar power density received on the earth is 1.4kW/m^2. See Problem 3.2 for other data.

3.4 Derive (3.5b) from (3.5a), assuming that $\mathbf{E} = E_x \hat{\mathbf{x}}$, and E_x is a function of z only.

3.5 The star α Centauri is approximately 4.33 light-years distant from Earth. A light-year is a unit of length that is the distance a light wave covers in one year. How distant is α Centauri in kilometers?

3.6 An electromagnetic pulse is sent from an earth station to the moon, and the reflected pulse is received 2.56 s later. How far is the moon from the Earth? (An electromagnetic pulse consists of a wide spectrum of electromagnetic waves at different frequencies.)

3.7 Find the SI units of the following quantities associated with a uniform electromagnetic wave: (a) ω, (b) k, (c) f, (d) T, and (e) λ.

3.8 A helium-neon laser emits light at a wavelength 6.328×10^{-7} m in air. Calculate its frequency, period, and wave number.

3.9 Figure P3.9 shows a dipole antenna. It is very effective in receiving television signals when its length is approximately equal to one-half the signal wavelength. What are approximate antenna lengths for receiving signals for the following: (a) Channel 2 ($f = 57$ MHz) and (b) Channel 13 ($f = 213$ MHz)?

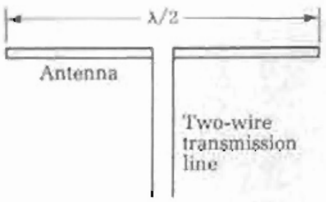


Figure P3-9

3.10 The following set of electromagnetic fields satisfies the time-harmonic Maxwell's equations in free space:

$$\mathbf{E} = E_0\, e^{+jkz}\, \hat{\mathbf{x}}$$

and

$$\mathbf{H} = H_0\, e^{+jkz}\, \hat{\mathbf{y}}$$

Express H_0 and k in terms of E_0 and ϵ_0 and μ_0.

3.11 Do the fields in the previous problem represent a uniform plane wave? In what direction does the wave travel? Find its velocity and determine the time-average Poynting vector $\langle \mathbf{S} \rangle$.

3.12 The Federal Communications Commission of the United States requires a minimum of 25 mV/m field intensity for AM stations covering the commercial area of a city. What is the power density associated with this minimum field? What is the intensity of the minimum magnetic field H?

3.13 Study the following **E** field in a source-free region:

$$\mathbf{E} = \hat{\mathbf{x}}\, E_0\, e^{-jkx}$$

Does it satisfy Maxwell's equations? If so, find the k and the **H** field. If not, explain why.

3.14 Show that in (3.13), if $\phi_a - \phi_b = \pi/2$ and $a = b$, the wave is right-hand circularly polarized.

3.15 Find the polarization (linear, circular, or elliptical and left-hand or right-hand) of the following fields:

(a) $\mathbf{E} = (j\hat{\mathbf{x}} + \hat{\mathbf{y}})\, e^{-jkz}$
(b) $\mathbf{E} = [(1 + j)\,\hat{\mathbf{y}} + (1 - j)\hat{\mathbf{z}}]\, e^{-jkx}$
(c) $\mathbf{E} = [(2 + j)\,\hat{\mathbf{x}} + (3 - j)\,\hat{\mathbf{z}}]\, e^{-jky}$
(d) $\mathbf{E} = (j\,\hat{\mathbf{x}} + j2\hat{\mathbf{y}})\, e^{+jkz}$

3.16 Show that, if $a = b$ and $\phi_a - \phi_b = \pi/4$, the wave is elliptically polarized. (Refer to (3.13).) Do not try to obtain an analytical expression for the locus. Just obtain a pair of parametric equations similar to (3.14), calculate E_x and E_y at ten points ($\omega t = 0, 10°, \ldots, 90°$), and sketch the locus.

3.17 Show that an elliptically polarized wave can be decomposed into two circularly polarized waves, one left-handed and the other right-handed. **Hint:** Let

$$\mathbf{E} = (a\hat{x} + b\hat{y})\, e^{-jkz},$$

where a and b are, in general, complex numbers. Then, let

$$\mathbf{E} = (a'\hat{x} + ja'\hat{y})\, e^{-jkz} + (b'\hat{x} - jb'\hat{y})\, e^{-jkz}$$

and solve for a′ and b′ in terms of a and b.

3.18 Show that a linearly polarized wave can be decomposed into two circularly polarized waves.

3.19 A dipole antenna is in the x-y plane and makes a 45° angle to the x axis. A receiver attached to the antenna is calibrated to read directly the component of the E field that is parallel to the dipole. What are the readings when the fields are those given in (a)–(d) of Problem 3.15?

3.20 An electromagnetic wave in vacuum has frequency f_0, wavelength λ_0, wave number k_0, and velocity v_0. When it enters a dielectric medium characterized by μ_0 and $\epsilon = 4\epsilon_0$, what are the f, λ, k, and v of the wave in this medium?

3.21 Aluminum has $\epsilon = \epsilon_0$, $\mu = \mu_0$, and $\sigma = 3.54 \times 10^7$ mho/m. If an antenna for UHF reception is made of wood coated with a layer of aluminum and if its thickness ought to be five times greater than the skin depth of the aluminum at that frequency, determine the thickness of the aluminum layer. Is ordinary aluminum foil thick enough for that purpose? Use $f = 1$ GHz. Ordinary aluminum foil is approximately 1/1000 in. thick.

3.22 Calculate the attenuation rate and skin depth of earth for a uniform plane wave of 10 MHz. Take the following data for the earth: $\mu = \mu_0$, $\epsilon = 4\epsilon_0$, and $\sigma = 10^{-4}$ mho/m.

3.23 Find the power density in earth where the field intensity is 1 V/m. Use the data in Problem 3.22.

3.24 Suppose that an airplane uses a radar to measure its altitude. Let the frequency of the radar be 3 GHz. Suppose further that the ground is covered with a meter of hard-packed snow.

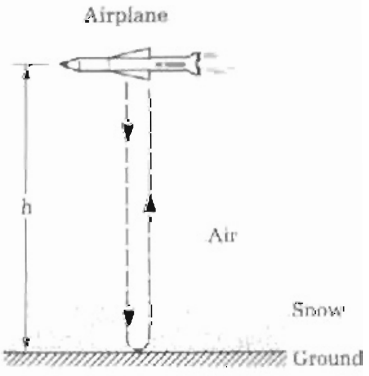


Figure P3.24

(a) What is the difference between the apparent altitude measured by the radar and the true altitude?

(b) How much attenuation in dB does the radar signal suffer because of the snow? Consider only the attenuation of the wave in the snow, and neglect the effect of snow on the reflection at air-snow and at snow-ground interfaces. Refer to Figure P3.24. Use $\epsilon = 1.5\epsilon_0$ and $\tan\delta = 9 \times 10^{-4}$ for hard-packed snow at 3 GHz.

3.25 The following data are given for a uniform plane wave in a dissipative medium:

(i) amplitude of E_x at $z = 0$ is 1 V/m,
(ii) phase of E_x at $z = 0$ is zero,
(iii) $k = 0.5 - j\,0.5$ (1/m),
(iv) direction of propagation is in $\hat{z}$,
(v) intrinsic impedance of the medium is $1 + j$ ohms

(a) Find the phasor expression for $\mathbf{E}_x$ as a function of z.
(b) Find the phasor expression for $\mathbf{H}$ as a function of z.
(c) Sketch E_x at $z = 0$ and at $z = 2$ m, as functions of ωt.
(d) Sketch the time-domain H fields at $z = 0$ and $z = 2$ m as functions of ωt.

3.26 Consider that a small space vehicle with 100 kg of mass is located in outer space where the gravitational field is negligible and the fuel has been exhausted. A searchlight of 1 kW is turned on, with hopes that the vehicle will gain some speed. How much speed will it finally gain if the searchlight can last 48 hours? Hint: The light wave carries radiation pressure, and there is a reaction force on the source of the light.

3.27 An ice particle of radius a is r distance away from the sun. The gravitational force acting on the particle is given by (3.46). The ice particle's mass can be obtained from its volume and its density which is assumed to be 1 gram/cm^3. The ice particle is also subject to radiation pressure which is given in (3.47). The force acting on the ice particle due to radiation pressure is approximately equal to the cross-sectional area of the particle times the radiation pressure. Show that, when the particle's radius is less than a critical value, the radiation force will be greater than the gravitational force, and this critical radius is independent of r, the distance from the sun. As a result, all particles with radii smaller than this critical radius tend to be blown out of the solar system. Find the value of this critical radius.

CHAPTER 3

3.1 power density $= 0.6\times10^{-22}$ W/m²-Hz

$\therefore$ power density in 54-60MHz $= (60-54)\times10^{6}\times0.6\times10^{-22} = 3.6\times10^{-16}$ W/m².

3.2 power density in the frequency range 54 - 60 MHz $= 0.6\times10^{-22}$ W/m²-Hz

$\therefore$ Total power density $= (60-54)\times10^{6}\times0.6\times10^{-22}\times4\pi\times(1.5\times10^{11})^2 = 101.8$ MW

3.3 $P = (1.4\times10^{3})\times4\pi\times(1.5\times10^{11})^2 = 4\times10^{26}$ W

3.4 From (3.5a) and $\underline{E} = E_x\hat{x}$, we have $\nabla^2 E_x + \omega^2\mu_0\varepsilon_0 E_x = 0$.

$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial z^2}$ because E_x is a function of z only. (3.5b) is then obtained.

3.5 3×10^{8} m/sec $\times 60\times60\times24\times365\times4.33 = 4.09\times10^{16}$ m $= 4.09\times10^{13}$ km.

3.6 3×10^{8} m/sec $\times (2.56/2)$ sec $= 3.84\times10^{8}$ m

3.7 (a) $[\omega]$ = rad/sec, (b) $[K] = 1/m$, (c) $[f] = 1/sec$, (d) $[T]$ = sec, (e) $[\lambda] = m$

3.8 $f = 3\times10^{8}/6.328\times10^{-7} = 4.74\times10^{14}$ Hz, $T = 1/f = 2.11\times10^{-15}$ sec, $k = 2\pi/\lambda = 9.93\times10^{6}$ (1/m)

3.9 (a) $3\times10^{8}/(2\times57\times10^{6}) = 2.63$ m. (b) $3\times10^{8}/(2\times213\times10^{6}) = 0.704$ m

3.10 $\nabla\times\underline{E} = -j\omega\mu_0\underline{H} \Rightarrow \underline{H} = \frac{\nabla\times\underline{E}}{j\omega\mu_0} = -\frac{1}{j\omega\mu_0}\nabla\times(E_0 e^{jkz}\hat{x}) = -\frac{k}{\omega\mu_0}E_0 e^{jkz}\hat{y} = H_0 e^{jkz}\hat{y}$

$\therefore H_0 = -\frac{k}{\omega\mu_0}E_0$ Also $\nabla^2\underline{E} + \omega^2\mu_0\varepsilon_0\underline{E} = 0 \Rightarrow k^2 = \omega^2\mu_0\varepsilon_0$ $\therefore k = \omega\sqrt{\mu_0\varepsilon_0}$ and $H_0 = -\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0$

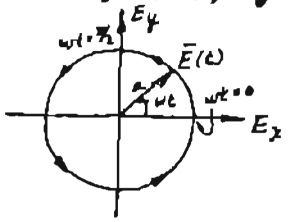
3.11 yes; $-z$ direction; $v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$; $\langle\underline{S}\rangle = \frac{1}{2}Re\{\underline{E}\times\underline{H}^*\} = \frac{1}{2}(E_0\times H_0)\hat{z} = -\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\hat{z}$

3.12 $|H| = 25\times10^{-3}/\sqrt{\frac{\mu_0}{\varepsilon_0}} = 25\times10^{-3}/120\pi = 6.63\times10^{-5}$ A/m, $|\langle S\rangle| = \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}|H|^2 = 0.83\ \mu W/m^2$

3.13 No. $\nabla\times\underline{E} = \nabla\times(E_0 e^{-jkx}\hat{x}) = 0 \Rightarrow \underline{H} = 0$ (no corresponding H-field exists).

3.14 If $\phi_a - \phi_b = \pi/2$ and $a = b$; $E_x = a\cos(\omega t - kz + \phi_a)$, $E_y = a\sin(\omega t - kz + \phi_a)$

$E_x^2 + E_y^2 = a^2$ and at $kz = \phi_a$ plane :



right-hand circularly polarized

3.15 (a) $\underline{E} = (j\hat{x} + \hat{y})e^{-jkz} = \hat{x}e^{-j(kz-\frac{\pi}{2})} + \hat{y}e^{-jkz} \Rightarrow \overline{E}(t) = -\hat{x}\sin(\omega t - kz) + \hat{y}\cos(\omega t - kz)$

Right-hand circularly polarized

(b) $\underline{E} = [(1+j)\hat{y} + (1-j)\hat{z}]e^{-jkx} \Rightarrow \overline{E}(t) = \hat{y}\sqrt{2}\cos(\omega t - kx + \frac{\pi}{4}) + \hat{z}\sqrt{2}\cos(\omega t - kx - \frac{\pi}{4})$

Right-hand circularly polarized

(c) $\underline{E} = [(2+j)\hat{x} + (3-j)\hat{z}]e^{-jky} \Rightarrow \overline{E}(t) = \hat{x}\sqrt{5}\cos(\omega t - ky + 0.46) + \hat{z}\sqrt{10}\cos(\omega t - ky - 0.32)$

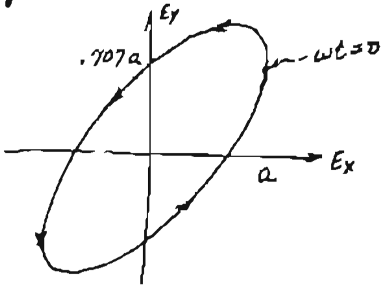
Left-hand elliptically polarized

(d) $\underline{E} = (j\hat{x} + j2\hat{y})e^{jkz} \Rightarrow \overline{E}(t) = -\hat{x}\sin(\omega t + kz) - \hat{y}\,2\sin(\omega t + kz)$

Linearly polarized.

3.16 If $a=b$, $\phi_a-\phi_b=\frac{\pi}{4}$; $E_x(t)=a\cos(\omega t-kz+\phi_a)$, $E_y=a\cos(\omega t-kz+\phi_a-\frac{\pi}{4})$
at $kz=\phi_a$, $E_x(t)=a\cos\omega t$ and $E_y=a\cos(\omega t-45°)$

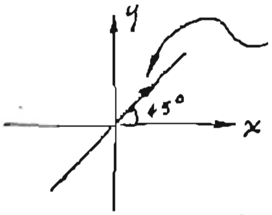
ωt	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
E_x	a	.985a	.94a	.866a	.766a	.643a	.5a	.342a	.174a	0
E_y	.707a	.819a	.906a	.966a	.996a	.996a	.966a	.906a	.819a	.707a



3.17 Elliptically polarized wave in general form : $\bar{E}=(\hat{x}\underline{a}+\hat{y}\underline{b})e^{-jkz}$, where $\underline{a}$ and $\underline{b}$ are complex numbers.
Let $\bar{E}=(\hat{x}\underline{a}'+\hat{y}j\underline{a}')e^{-jkz}+(\hat{x}\underline{b}'-\hat{y}j\underline{b}')e^{-jkz}$
=(Left-hand circularly polarized)+(Right-hand circularly polarized)
Then $(\hat{x}\underline{a}+\hat{y}\underline{b})e^{-jkz}=(\hat{x}\underline{a}'+\hat{y}j\underline{a}')e^{-jkz}+(\hat{x}\underline{b}'-\hat{y}j\underline{b}')e^{-jkz}$
$\therefore\ \underline{a}'+\underline{b}'=\underline{a}$ and $j(\underline{a}'-\underline{b}')=\underline{b}$ $\Rightarrow$ $\underline{a}'=\frac{1}{2}(\underline{a}-j\underline{b})$ and $\underline{b}'=\frac{1}{2}(\underline{a}+j\underline{b})$
Hence, decomposed waves are : $[\hat{x}\frac{1}{2}(\underline{a}-j\underline{b})+\hat{y}\frac{1}{2}(j\underline{a}+\underline{b})]e^{-jkz}$ (left-handed)
and $[\hat{x}\frac{1}{2}(\underline{a}+j\underline{b})-\hat{y}\frac{1}{2}(j\underline{a}-\underline{b})]$ (right-handed)

3.18 Linearly polarized wave in general form : $\bar{E}=(\hat{x}a+\hat{y}b)e^{-jkz}$, where a and b are real numbers.
Let $\bar{E}=(\hat{x}a+\hat{y}b)e^{-jkz}=(\hat{x}\underline{a}'+\hat{y}j\underline{a}')e^{-jkz}+(\hat{x}\underline{b}'-\hat{y}j\underline{b}')e^{-jkz}$, then
$\underline{a}'+\underline{b}'=a$ and $j(\underline{a}'-\underline{b}')=b$ $\Rightarrow$ $\underline{a}'=\frac{1}{2}(a-jb)$ and $\underline{b}'=\frac{1}{2}(a+jb)$
$\therefore$ Decomposed waves are $[\hat{x}\frac{1}{2}(a-jb)+\hat{y}\frac{1}{2}(ja+b)]e^{-jkz}$ (left-handed) and
$[\hat{x}\frac{1}{2}(a+jb)-\hat{y}\frac{1}{2}(ja-b)]$ (right-handed).

3.19



unit vector $\hat{a}=\hat{x}\frac{1}{\sqrt{2}}+\hat{y}\frac{1}{\sqrt{2}}$

(a) $\bar{E}\cdot\hat{a}=(j\hat{x}+\hat{y})e^{-jkz}\cdot(\frac{\hat{x}}{\sqrt{2}}+\frac{\hat{y}}{\sqrt{2}})=\frac{1}{\sqrt{2}}(1+j)e^{-jkz}=1e^{-j(kz-\frac{\pi}{4})}$
$\therefore|\bar{E}\cdot\hat{a}|=1$

(b) $\bar{E}\cdot\hat{a}=[-\frac{(1+j)}{\sqrt{2}}]e^{-jkz}$ $\quad\therefore|\bar{E}\cdot\hat{a}|=\frac{\sqrt{2}}{\sqrt{2}}=1$
(c) $\bar{E}\cdot\hat{a}=[\frac{2+j}{\sqrt{2}}]e^{-jkz}$ $\quad\therefore|\bar{E}\cdot\hat{a}|=\sqrt{2.5}=1.58$
(d) $\bar{E}\cdot\hat{a}=[\frac{j+j2}{\sqrt{2}}]e^{-jkz}$ $\quad\therefore|\bar{E}\cdot\hat{a}|=\frac{3}{\sqrt{2}}=2.12$

3.20 $f = f_o,\ v = \frac{1}{\sqrt{\mu_o(4\varepsilon_o)}} = \frac{v_o}{2},\quad \lambda = v/f = \frac{1}{2}(v_o/f_o) = \lambda_o/2,\quad k = \frac{2\pi}{\lambda} = 2\left(\frac{2\pi}{\lambda_o}\right) = 2k_o$

3.21 $\sigma/\omega\varepsilon = \frac{3.54\times10^7}{3\pi\times10^9}\times36\pi\times10^9 >> 1 \quad \therefore\ \delta \simeq \sqrt{\frac{1}{\pi f\mu_o\sigma}} = \left(\frac{10^7}{\pi\times10^9\times4\pi\times3.54\times10^7}\right)^{1/2} = 2.67\times10^{-6}\,m$

$5\delta = 1.34\times10^{-5}\,m < 2.5\times10^{-5}\,m$ $\therefore$ aluminum foil is thick enough.

3.22 $\sigma/\omega\varepsilon = \frac{10^{-4}}{2\pi\times10^7}\times\frac{36\pi\times10^9}{4} = 4.5\times10^{-2} << 1$

$k_I = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2}\sqrt{\frac{\mu_o}{4\varepsilon_o}} = \frac{\sigma}{4}\times120\pi = 10^{-4}\times30\pi = 0.0094\,(1/m) \quad \therefore\ d_p = \frac{1}{k_I} = 106.1\,(m)$

3.23 $\eta = \sqrt{\frac{\mu}{\varepsilon(1-j\sigma/\omega\varepsilon)}} = \frac{1}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\sqrt{\frac{1}{1-j\sigma/\omega\varepsilon}} = \frac{1}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}}(1-j0.045)^{-1/2} \sim \frac{1}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}} = 60\pi$

$\therefore$ At $z=0$, $|\langle S\rangle| = \frac{1}{2}\frac{E_o^2}{|\eta|}e^{j\phi} \sim \frac{1}{2}\frac{E_o^2}{60\pi} = 2.65\,mW/m^2$

3.24 (a) Let v_{air} = velocity in air, then $t = 2\left[\frac{(h-1)}{v_{air}} + \frac{1}{(v_{air}/\sqrt{1.5})}\right] = 2\left(\frac{h_{app}}{v_{air}}\right)$

$\therefore\ h - 1 + \sqrt{1.5} = h_{app} \Rightarrow h_{app} - h = \sqrt{1.5} - 1 = 0.225\,m$ (h_{app} = apparent altitude)

(b) $\underline{k} = \omega\sqrt{\mu_o(1.5\varepsilon_o)}(1-j9\times10^{-4})^{1/2} \sim \omega\sqrt{\mu_o(1.5\varepsilon_o)}(1-j4.5\times10^{-4}) \Rightarrow k_I = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{1.5}(4.5\times10^{-4})\left(\frac{Nepr}{m}\right)$

Attenuation due to snow layer : $k_I\times2(m) = \frac{2\pi\times3\times10^9}{3\times10^8}\sqrt{1.5}\times2\times4.5\times10^{-4} = 0.069\,(Neper)$

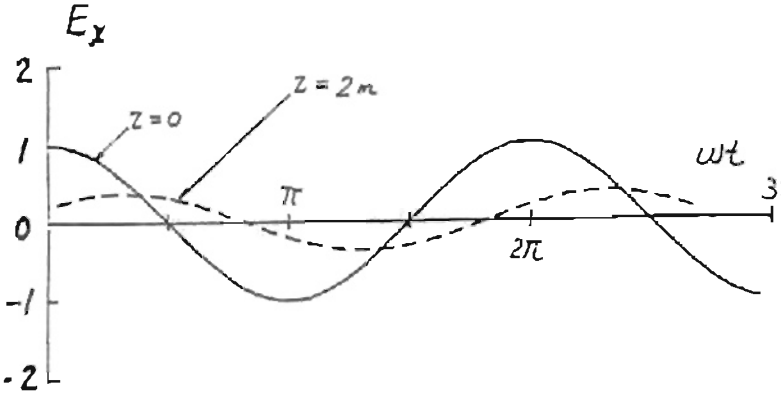
$= 0.069\times8.686 = 0.6\,(dB)$

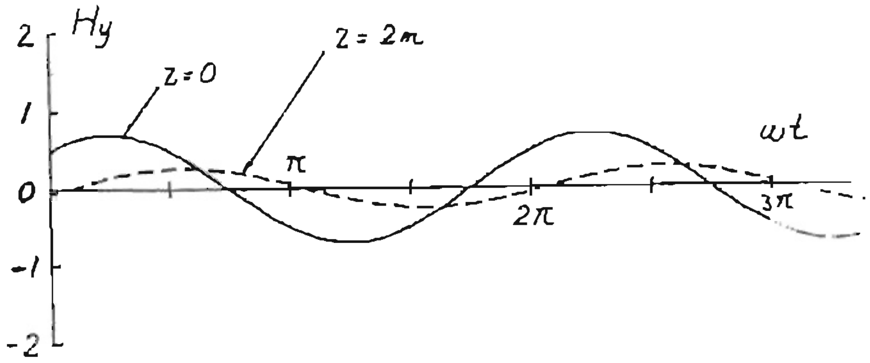
3.25 (a) $E_x = \exp[-j(0.5-j0.5)z] = \exp(-0.5z)\exp(-j0.5z)$

(b) $\underline{H} = \hat{y}\frac{1}{1+j}\exp(-0.5z)\exp(-j0.5z) = \hat{y}(0.5-j0.5)\exp(-0.5z)\exp(-j0.5z)$

(c) $E_x(z,t) = \exp(-0.5z)\cos(\omega t - 0.5z)$

(d) $H_y(z,t) = 0.707\exp(-0.5z)\cos(\omega t - 0.5z - \pi/4)$





3.26 $F = \frac{\langle S \rangle A}{c} = \frac{10^3}{3 \times 10^8}$; $a = \frac{F}{m} = \frac{10^3}{3 \times 10^8 \times 10^2} = \frac{1}{3 \times 10^7}$

$v = at = \frac{1}{3 \times 10^7} \times 48 \times 60 \times 60 = 5.8 \times 10^{-3}$ m/s

3.27 $F_g = 6.67 \times 10^{-11} \frac{m_1 m_2}{r^2}$; $F_r = 1.4 \times 10^3 \frac{\pi a^2}{c} \frac{r_e^2}{r^2}$

$F_g = F_r \rightarrow 6.67 \times 10^{-11} m_1 m_2 = 1.4 \times 10^3 \frac{\pi a^2}{c} r_e^2$

$m_1 = \frac{4}{3} \pi a^3 \cdot 10^6 g = \frac{4}{3} \pi a^3 \times 10^3 kg$

$m_2 = 1.99 \times 10^{30} kg$; $r_e = 1.5 \times 10^{11} m$

$6.67 \times 10^{-11} \frac{4}{3} \pi a^3 \times 10^3 m_2 = 1.4 \times 10^3 \frac{\pi a^2}{c} \times r_e^2$

$$a = \frac{1.4 \times 10^3 \times (1.5)^2 \times 10^{22} \times 3}{3 \times 10^8 \times 6.67 \times 10^{-11} \times 4 \times 10^3 \times 1.99 \times 10^{30}} = 0.59 \times 10^{-6} m$$

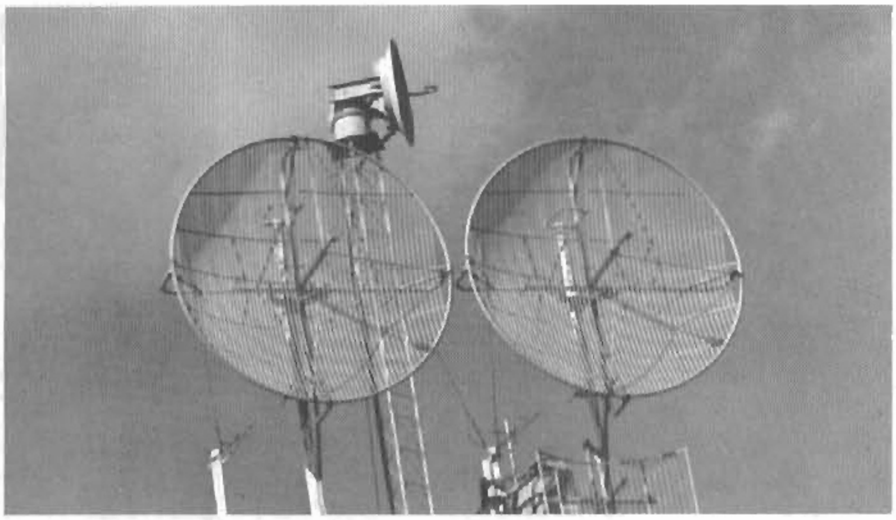


Figure 4.23 To receive linearly polarized electromagnetic waves, wire gratings may replace metal plates for reflector antennas.

Thus,

$$\mathbf{J}_s = \hat{y}\left(\frac{2E_0}{\eta}\right)\cos\theta\, e^{-jkx\sin\theta} \tag{4.54}$$

Note that the current flows in the $\hat{y}$ direction and that no current flows in the $\hat{x}$ direction. In fact, if the conducting plate is replaced by a grating of parallel conducting wires arranged in the $\hat{y}$ direction, these wires also serve as a reflector that is as effective as a solid conducting plate. Experiments have found that grates are effective when the spacing of the wire in the grate is much smaller than the wavelength of the wave. Grates are used instead of conducting plates to reflect linearly polarized electromagnetic waves because they reduce weight, save material, and decrease resistance to wind. Based on these considerations, some reflectors use wires to replace metal dishes for transmitting and receiving electromagnetic waves. An example of such a structure is shown in Figure 4.23.

Problems

4.1 The **E** field measured in air just above a glass plate is equal to 2 V/m in magnitude and is direct at 45° away from the boundary, as shown in Figure P4.1. The magnitude of the **E** field measured just below the boundary is equal to 3 V/m. Find the angle θ for the **E** field in the glass just below the boundary.

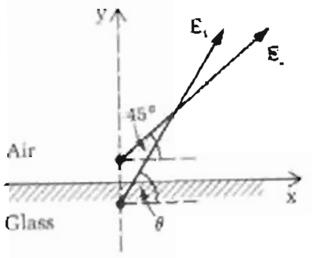


Figure P4.1

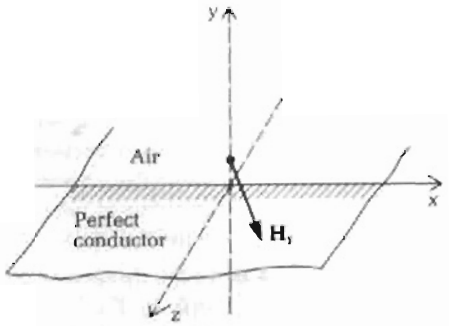


Figure P4.2

4.2 The H field in air just above a perfect conductor is given by

$$H_1 = 3\hat{x} + 4\hat{z} \text{ amperes per meter}$$

as shown in Figure P4.2. Find the surface current J_s on the surface of the perfect conductor. The conductor occupies the space $y < 0$.

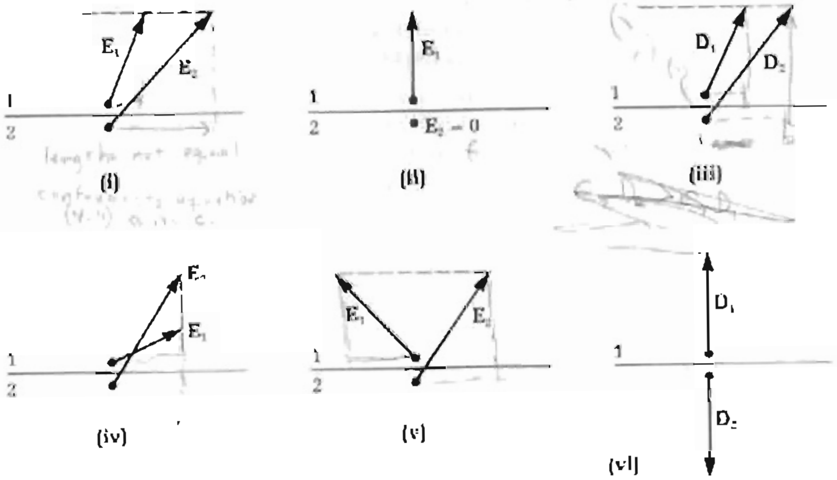
4.3 Match the following descriptions with the figures shown in Figure P4.3. Fields are near the interface but on opposite sides of the boundary.

(a) medium 1 and medium 2 are dielectrics with $\epsilon_1 > \epsilon_2$
(b) medium 1 and medium 2 are dielectrics with $\epsilon_1 < \epsilon_2$
(c) impossible
(d) impossible
(e) there is a positive surface charge on the boundary between two dielectrics
(f) medium 2 is a perfect conductor

$D_{2tan} > D_{1tan}$
$\epsilon E_2 > \epsilon E_1$

E → Tangential
D → Normal

Figure P4.3



lengths not equal

(i)

contradicting equation (4.4) a.i.c.

(ii)

(iii)

$D_{2tan} > D_{1tan}$
$\epsilon_2 E_2 > \epsilon_1 E_1$

(iv)

(v)

(vi)

4.4 Calculate the critical angle θ_c of an air-glass interface similar to the interface shown in Figure 4.8. The dielectric constant of glass at optical frequencies is 2.25 times that of air.

4.5 A pearl is embedded at the middle of a cubic heavy-lead glass ($\epsilon_r = 3.6$). Is it possible to cover a **portion** of the surface of the cube so that from outside the pearl will not be seen at any viewing angle? If so, find the shape and the minimum area of the cover (in terms of the cubic surface area). **Hint:** consider conditions of total reflection, and neglect multiple internal reflections.

4.6 In the three-media configuration shown in Figure P4.6, the wave numbers are k_1, k_2, and k_3. Find the transmission angle in medium 3 in terms of θ_1 and the wave numbers. Assume all k's are real.

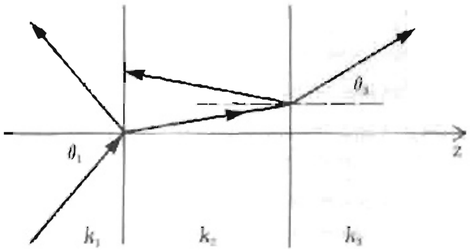


Figure P4.6

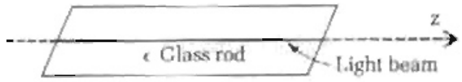


Figure P4.7

4.7 Solid-state lasers (ruby or glass) are often fabricated of rods with the ends bevelled at the Brewster angle. Let $\epsilon = 2\epsilon_0$ for the rod. Sketch the proper arrangement of the external mirrors and their angles. Indicate the bevelled angle of the glass rod. What is the polarization of the output of the laser beam? (See Figure P4.7.)

4.8 A parallel-polarized wave is incident from medium 1 on the plane boundary between medium 1 and medium 2. Both media are dielectrics with $\mu_1 = \mu_2 = \mu_0$ and real permittivities ϵ_1 and ϵ_2. We know that, when the incident angle is larger than the critical angle θ_c, no time-average power is transferred to medium 2. Also, when the incident angle is equal to the Brewster angle θ_b, the reflected power is zero. Now imagine a situation in which the Brewster angle is greater than the critical angle. A wave incident at the Brewster angle will not be reflected, because the incident angle is equal to θ_b, nor will it be transmitted, because the incident angle is greater than θ_c. Is this situation possible? Why?

4.9 Consider an electromagnetic wave of 1 MHz impinging at 60° on the ionosphere. This case is similar to that shown in Figure 4.13. Assume the the plasma frequency of the ionosphere is $\omega_p = 2\pi \times 9 \times 10^6$ rad/s, and plot $|\mathbf{E}|$ as a function of z (like in Figure 4.14). Mark the scale of z in meters. Solve only for the case of parallel polarization with $E_0 = 1$ V/m.

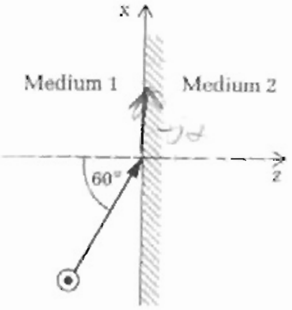


Figure P4.10

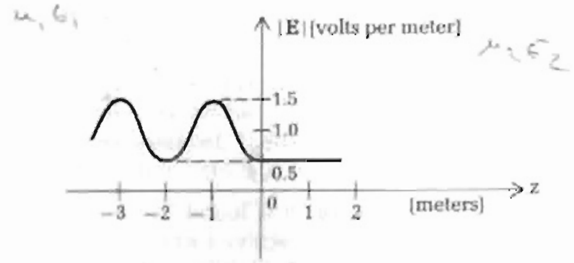


Figure P4.11

4.10 A perpendicularly polarized electromagnetic wave impinges from medium 1 (characterized by $\mu_1 = \mu_0$ and $\epsilon_1 = 4\epsilon_0$) to medium 2 (characterized by $\mu_2 = \mu_0$ and $\epsilon_2 = \epsilon_0$). This situation is shown in Figure P4.10. $k = \omega\sqrt{\mu\epsilon}$

(a) What is the critical angle? $\theta_c = \sin^{-1}\frac{k_2}{k_1} = \sin^{-1}\frac{1}{2} = 30°$

(b) Let the incident angle be 60°; find k_x and k_z in terms of $k_0 = \omega\sqrt{\mu_0\epsilon_0}$.

(c) Find k_{tz} in terms of k_0.

(d) In the second medium, find the distance z_0 at which the field strength is $1/e$ of that at $z = 0^+$.

(e) Find $|R_\perp|$ and the phase shift $\arg(R_\perp)$.

? **4.11** A uniform plane wave in air impinges normally on a dielectric wall. The magnitude of the total **E** field measured in front of the wall is shown in Figure P4.11.

(a) What is the permittivity of the dielectric wall? Assume $\mu_2 = \mu_0$.

(b) What is the frequency of the wave?

4.12 A uniform plane wave in air impinges on a lossless dielectric material at a 45° angle, as shown in Figure P4.12. The transmitted wave propagates in a 30° direction with respect to the normal. The frequency is 300 MHz.

(a) Find ϵ_2 in terms of ϵ_0.

(b) Find the reflection coefficient $R_{//}$.

(c) Obtain the mathematical expressions for the incident **E** field, the reflected **E** field, and the transmitted **E** field.

(d) In both media, sketch the standing wave pattern of $|\mathbf{E}_{x\,\text{total}}|$ as a function of z.

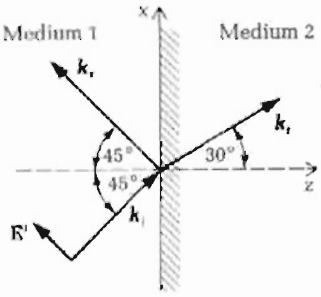


Figure P4.12

4.13 For two isotropic media with $\mu_1 \neq \mu_2$ and $\epsilon_1 \neq \epsilon_2$, find the Brewster angle for both the perpendicular polarization and the parallel polarization.

4.14 If a wire antenna is attached parallel to the metallic surface of a vehicle and is insulated from the surface by a thin layer of dielectric material with a thickness approximately equal to 1 mm, would it receive an AM signal ($f = 1$ MHz)? **Hint:** Wire antenna interacts only with E field in the direction of the wire.

4.15 It is known that the transmitting antenna of an FM station is located in the direction perpendicular to a metallic plate, as shown in Figure 4.15. The frequency of the signal is 94 MHz.

(a) Where should a receiving antenna be placed to receive maximum signal? The antenna is a dipole that interacts with the E field.

(b) If the amplitude of the incident E field is 1 V/m, what is the amplitude of the E field at this optimum position?

4.16 It is found that by placing a conducting plate 0.8 m behind a dipole antenna, the received signal coming from the normal direction is twice as strong as the incident field. What is the frequency of the signal? What would the strength of the total E field be if the frequency of the wave is changed to 98 MHz while the antenna is still placed 0.8 m from the plate?

4.17 What would the E field be if the receiving system in Problem 4.15 were to detect a wave coming in at an angle 10° off the normal? Assume that all other parameters remain the same.

4.18 Derive (4.53).

4.19 For a parallel-polarized uniform plane wave impinging on a perfect conductor at an angle θ, find the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ for the incident and for the reflected waves.

4.20 Consider a 90° "corner reflector" shown in Figure P4.20. It is made of two conducting plates placed perpendicularly to each other. A uniform plane wave with $\mathbf{E} = \hat{z}E_0 \exp(jkx \cos\theta + jky \sin\theta)$ impinges on the structure at an angle θ. Show that the total electric field is $\mathbf{E} = -\hat{z}4E_0 \sin(kx \cos\theta) \sin(ky \sin\theta)$. **Hint:** The field is the sum of four waves with four $\boldsymbol{k}$-vectors shown in Figure P4.20.

4.21 Use the formula given in Problem 4.20 for the total electric field. Find the optimum position of a dipole antenna placed in front of the 90° corner reflector. The θ angle of the incident wave is 30°. The frequency is 100 MHz. Express the position in $x - y$ coordinates in meters. What is the "gain" of this receiving antenna? Gain is defined as follows:

$$\text{Gain} = 20 \log_{10} \left| \frac{E_\alpha}{E_0} \right| \qquad \text{(dB)}$$

where E_α is the E field at the antenna position and E_0 is the field strength of the incident wave.

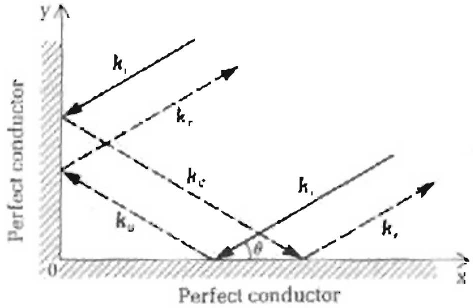
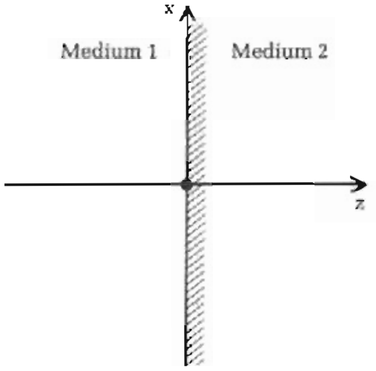
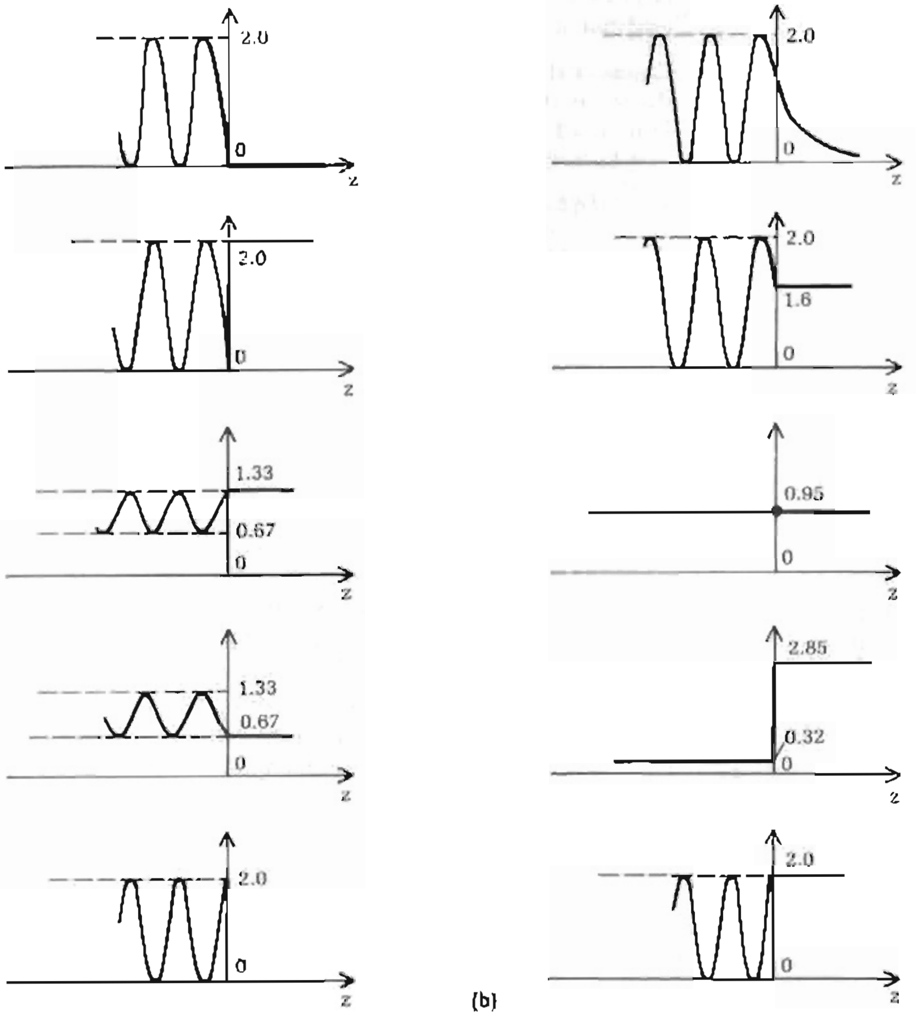


Figure P4.20 Top view of a 90° corner reflector and the four $\boldsymbol{k}$-vectors.



(a)

Figure P4.22



(b)

Figure P4.22

4.22 Match the following descriptions to the standing wave patterns shown in Figure P4.22. The incident wave in medium 1 has an amplitude equal to 1 V/m. Note: There are three patterns that do not fit any of the following descriptions. Cross out these patterns.

(i) Plot of $|E_{y\ \text{total}}|$, with medium 1 being air, medium 2 having $\epsilon_2 = 4\epsilon_0$ and $\mu_2 = \mu_0$. Normal incidence.
(ii) Plot of $|E_{y\ \text{total}}|$, with medium 1 being characterized by $\epsilon_1 = 4\epsilon_0$ and $\mu_1 = \mu_0$, and medium 2 being air. Normal incidence.
(iii) Plot of $|E_{y\ \text{total}}|$, with medium 1 being characterized by $\epsilon_1 = 4\epsilon_0$ and $\mu_1 = \mu_0$, and medium 2 being air. The incidence angle is greater than the critical angle.
(iv) Plot of $|E_{x\ \text{total}}|$, incidence angle is equal to the Brewster angle.
(v) Plot of $|E_{z\ \text{total}}|$, incidence angle is equal to the Brewster angle. ϵ_1 is greater than ϵ_2.
(vi) Plot of $|E_{y\ \text{total}}|$. Medium 1 is air and medium 2 is perfect conductor.
(vii) Plot of $|H_{y\ \text{total}}|$ (η_1). Medium 1 is air and medium 2 is perfect conductor.

4.23 Consider the case of normal incidence of a uniform plane wave on a perfect conductor as shown in Figure 4.15. It can be seen in (4.47) that an oscillating current is induced on the surface of the conductor. Therefore, the following expression may be written for the velocity of a charge on the conductor:

$$\mathbf{v} = \hat{x}\, d\, q\, E_0 \cos(\omega t - kz)$$

The above equation is exactly the same as Equation (3.39). Continue to work along this line and prove that the time-average radiation pressure on the perfect conductor is two times that given in (3.45).

CHAPTER 4

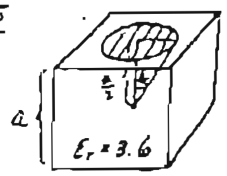
4.1 $E_{1\tan} = E_{2\tan} \Rightarrow E_1\cos\theta = E_2\cos 45^\circ$ ∴ $\cos\theta = \frac{E_2}{E_1}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{3}$ ∴ $\theta = 61.9^\circ$

4.2 $\bar{J}_s = \hat{y}\times\bar{H}_1 = \hat{y}\times(3\hat{x}+4\hat{z}) = 4\hat{x}-3\hat{z}$ (A/m)

4.3 (i) c (ii) f (iii) b (iv) a (v) d (vi) e

4.4 $\theta_c = \sin^{-1}\sqrt{1/2.25} = 41.8^\circ$

4.5



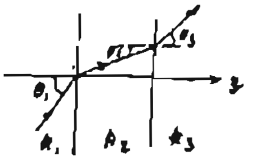
$\sin\theta_c = \sqrt{1/3.6}$, cover portion on each surface is a circle.

Total cover area $= 6\times\pi\times\left(\frac{a}{2}\tan\theta_c\right)^2 = \frac{3\pi}{2}a^2\,\frac{\sin^2\theta_c}{1-\sin^2\theta_c} = 1.812\,a^2$

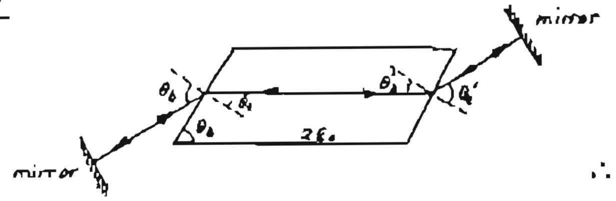
Cubic area $= 6\times a^2 = 6a^2$

% of cover area $= \frac{1.812}{6}\times 100\% = 30.2\%$

4.6 Snell's law: $k_1\sin\theta_1 = k_2\sin\theta_2 = k_3\sin\theta_3 \Rightarrow \theta_3 = \sin^{-1}\left(\frac{k_1}{k_3}\sin\theta_1\right)$



4.7



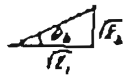
$$\begin{cases} n_1\sin\theta_b = n_2\sin\theta_t \\ \theta_b+\theta_t = \frac{\pi}{2}, \text{ if } \theta_b \text{ is the Brewster angle} \end{cases}$$

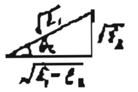
∴ $\sin\theta_b = \sqrt{2}\sin\left(\frac{\pi}{2}-\theta_b\right) = \sqrt{2}\cos\theta_b$, $\theta_b = \tan^{-1}\sqrt{2} = 54.7^\circ$

∴ $\theta_b = \theta_t' = 54.7^\circ$ and $\theta_t = \theta_b' = 90^\circ - 54.7^\circ = 35.3^\circ$

4.8 impossible; Brewster angle (θ_b) always less than critical angle (θ_c):

$\tan\theta_b = \sqrt{\varepsilon_2/\varepsilon_1}$ $\sin\theta_c = \sqrt{\varepsilon_2/\varepsilon_1}$





∴ $\theta_c > \theta_b$

4.9 $\mu_1 = \mu_0$, $\varepsilon_1 = \varepsilon_0$; $\mu_2 = \mu_0$. $\varepsilon_2 = \varepsilon_0\left[1-\left(\frac{2\pi\times 9\times 10^6}{2\pi\times 10^6}\right)^2\right] = -80\varepsilon_0$

$k_1 = k_0$, $\underline{k_2} = -j\sqrt{80}\,k_0$, $k_x = k_1\sin 60^\circ = \frac{\sqrt{3}}{2}k_0$, $k_z = k_1\cos 60^\circ = \frac{1}{2}k_0$, $\underline{k_{tz}} = -j\sqrt{80.75}\,k_0$

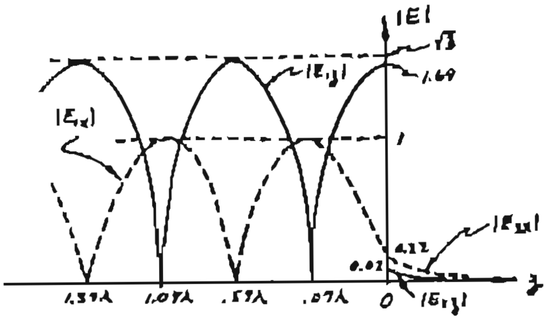
$\underline{R_\parallel} = \frac{(-80)\frac{1}{2}+j\sqrt{80.75}}{(-80)\frac{1}{2}-j\sqrt{80.75}} = e^{-j25.3^\circ}$, $\underline{T_\parallel} = 1+\underline{R_\parallel} = 1+e^{-j25.3^\circ}$

$|E_{1x}| = H_0\left(\frac{k_z}{\omega\varepsilon_0}\right)\left|1-\underline{R_\parallel}\,e^{j2k_z z}\right| = H_0\left(\frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\left|1-e^{-j25.3^\circ}e^{jk_0 z}\right| = \frac{1}{2}\left|1-e^{-j25.3^\circ}e^{jk_0 z}\right|$ (let $H_0\sqrt{\frac{\mu_0}{\varepsilon_0}} = 1$)

$|E_{1z}| = H_0\left(\frac{k_x}{\omega\varepsilon_0}\right)\left|1+\underline{R_\parallel}\,e^{j2k_z z}\right| = H_0\left(\frac{\sqrt{3}}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\left|1+e^{-j25.3^\circ}e^{jk_0 z}\right| = \frac{\sqrt{3}}{2}\left|1+e^{-j25.3^\circ}e^{jk_0 z}\right|$

$|E_{2x}| = H_0\left|\frac{k_{tz}}{\omega\varepsilon_0(-80)}\right|\left|\underline{T_\parallel}\right|e^{-\sqrt{80.75}k_0 z} = H_0\left(\frac{\sqrt{80.75}}{80}\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\left|1+e^{-j25.3^\circ}\right|e^{-\sqrt{80.75}k_0 z} = \frac{\sqrt{80.75}}{80}\left|1+e^{-j25.3^\circ}\right|e^{-\sqrt{80.75}k_0 z}$

$|E_{2z}| = H_0\left[\frac{k_x}{\omega\varepsilon_0(-80)}\right]\left|\underline{T_\parallel}\right|e^{-\sqrt{80.75}k_0 z} = H_0\left(\frac{\sqrt{3}}{160}\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\left|1+e^{-j25.5^\circ}\right|e^{-\sqrt{80.75}k_0 z} = \frac{\sqrt{3}}{160}\left|1+e^{-j25.3^\circ}\right|e^{-\sqrt{80.75}k_0 z}$

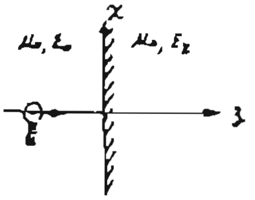


4.10 (a) $\sin\theta_c = \sqrt{1/4} = 1/2$ ∴ $\theta_c = 30^\circ$

(b) $k_x = k_1\sin 60^\circ = 2k_0\frac{\sqrt{3}}{2} = \sqrt{3}\,k_0$, $k_z = k_1\cos 60^\circ = 2k_0\frac{1}{2} = k_0$

(c) $k_{tz} = (k_t^2-k_x^2)^{1/2} = (k_0^2-3k_0^2)^{1/2} = -j\sqrt{2}\,k_0$ (d) $z_0 = 1/(\sqrt{2}\,k_0)$

(e) $\underline{R_\perp} = \frac{1+j\sqrt{2}}{1-j\sqrt{2}} = 1\,e^{j109.5^\circ}$



4.11 (a) $k_1 = \omega\sqrt{\mu_0\epsilon_0}$, $k_x = 0$, $k_z = k_1$; $k_2 = \omega\sqrt{\mu_0\epsilon_2}$, $k_{tz} = \sqrt{k_2^2 - k_x^2} = k_2$

Normal impinging : $\bar{E}_1^i = \hat{y} E_0 e^{-jk_z z}$, $\bar{E}_1^r = \hat{y} R_I E_0 e^{jk_z z}$

$R_I = \dfrac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}} = \dfrac{\sqrt{\epsilon_0} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_0} + \sqrt{\epsilon_2}}$, $|E_1^{total}| = |E_0|\,|1 + R_I e^{j2k_z z}|$

Take $|E_0| = 1$; at $z = 0$, $|E_1^{total}| = 0.5 = \left|1 + \dfrac{\sqrt{\epsilon_0} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_0} + \sqrt{\epsilon_2}}\right| \Rightarrow 4\sqrt{\epsilon_0} = \sqrt{\epsilon_0} + \sqrt{\epsilon_2}$ $\therefore \epsilon_2 = 9\epsilon_0$

(b) $|E|_{max}$ occurs at $z = -1m \Rightarrow e^{-j2k_1}$ must be equal to -1 (since $R_I < 0$)

$\therefore 2k_1 = \pi$ or $\omega = \dfrac{\pi}{2\sqrt{\mu_0\epsilon_0}} \Rightarrow f = \dfrac{3}{4}\times 10^8 = 0.75\times 10^8$ Hz

4.12 (a) $n_1 \sin 45^\circ = n_2 \sin 30^\circ \Rightarrow \epsilon_2 = 2\epsilon_0$

(b) $k_1 = k_0$, $k_x = \frac{1}{\sqrt{2}}k_0$, $k_z = \frac{1}{\sqrt{2}}k_0$; $k_2 = \sqrt{2}k_0$, $k_{tz} = \sqrt{k_2^2 - k_x^2} = \sqrt{\frac{3}{2}}k_0$

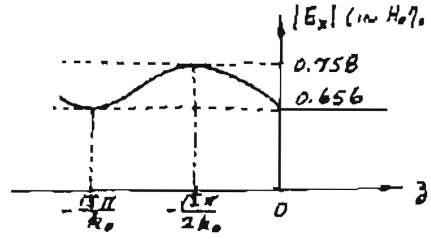
$R_{II} = \dfrac{2 - \sqrt{3}}{2 + \sqrt{3}} = 0.072$, $T_{II} = 1 + R_{II} = 1.072$

(c) $\bar{E}^i = \frac{1}{\sqrt{2}}(\hat{x} - \hat{z})(H_0\eta_0)e^{-j\frac{1}{\sqrt{2}}(k_0x + k_0z)}$

$\bar{E}^r = \frac{0.072}{\sqrt{2}}(-\hat{x} - \hat{z})(H_0\eta_0)e^{-j\frac{1}{\sqrt{2}}(k_0x - k_0z)}$

$\bar{E}^t = \frac{0.536}{\sqrt{2}}(\sqrt{3}\hat{x} - \hat{z})(H_0\eta_0)e^{-j\frac{1}{\sqrt{2}}(k_0x + \sqrt{3}k_0z)}$

(d) $|E_{1x}|^{total} = \frac{1}{\sqrt{2}}(H_0\eta_0)\left|1 - 0.072\,e^{j\sqrt{2}k_0z}\right|$; $|E_{2x}|^{total} = \frac{\sqrt{3}}{\sqrt{2}}(0.536)(H_0\eta_0) = 0.656\,(H_0\eta_0)$



4.13 $R_I = 0 \Rightarrow \mu_2 k_z = \mu_1 k_{tz} \Rightarrow \mu_2 k_1 \cos\theta_b = \mu_1\sqrt{k_2^2 - k_1^2\sin^2\theta_b}$

$\therefore \mu_2^2 k_1^2 \cos^2\theta_b = \mu_1^2 k_2^2 - \mu_1^2 k_1^2 \sin^2\theta_b = \mu_1^2 k_2^2 - \mu_1^2 k_1^2 + \mu_1^2 k_1^2 \cos^2\theta_b$

$\therefore \cos^2\theta_b = \dfrac{\mu_1^2 k_2^2 - \mu_1^2 k_1^2}{\mu_2^2 k_1^2 - \mu_1^2 k_1^2} = \dfrac{\mu_1(\mu_2\epsilon_2 - \mu_1\epsilon_1)}{\epsilon_1(\mu_2^2 - \mu_1^2)}$ $\therefore \theta_b = \cos^{-1}\sqrt{\dfrac{\mu_1(\mu_2\epsilon_2 - \mu_1\epsilon_1)}{\epsilon_1(\mu_2^2 - \mu_1^2)}}$

$R_{II} = 0 \Rightarrow \epsilon_2 k_z = \epsilon_1 k_{tz} \Rightarrow \epsilon_2^2 k_1^2 \cos^2\theta_b = \epsilon_1^2 k_2^2 - \epsilon_1^2 k_1^2 \sin^2\theta_b = \epsilon_1^2 k_2^2 - \epsilon_1^2 k_1^2 + \epsilon_1^2 k_1^2 \cos^2\theta_b$

$\therefore \cos^2\theta_b = \dfrac{\epsilon_1^2 k_2^2 - \epsilon_1^2 k_1^2}{\epsilon_2^2 k_1^2 - \epsilon_1^2 k_1^2} = \dfrac{\epsilon_1(\mu_2\epsilon_2 - \mu_1\epsilon_1)}{\mu_1(\epsilon_2^2 - \epsilon_1^2)}$ $\therefore \theta_b = \cos^{-1}\sqrt{\dfrac{\epsilon_1(\mu_2\epsilon_2 - \mu_1\epsilon_1)}{\mu_1(\epsilon_2^2 - \epsilon_1^2)}}$

4.14 $f = 10^6$ Hz $\Rightarrow \lambda = 300\,m$, $d = 1mm = (10^{-3}/300)\lambda \approx 3.33\times 10^{-6}\lambda$

$E_{tan} \approx 2E_0\sin(kd) = 2E_0\sin(\frac{2\pi}{\lambda}\times 3.33\times 10^{-6}\lambda) \approx 4.2\times 10^{-5}E_0$

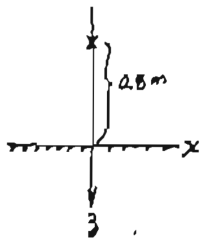
E_0 being the field in air in absence of the car.

4.15 (a) $f = 94$ MHz $\Rightarrow \lambda = 3.19\,m$ $\therefore$ receiving antenna should be located at $z = -\frac{\lambda}{4} = -80cm$

(b) $|E|^{total} = 2|E_0| = 2$ V/m

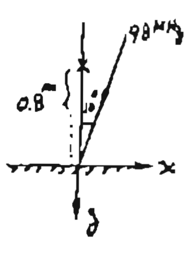
4.16 $|E| = 2|E_0|\sin(kz)$, $z = 0.8m$, $k = \dfrac{2\pi\times 98\times 10^6}{3\times 10^8} = 2.05$

$\therefore |E| = |E_0|\,2\sin(2.05\times 0.8) = 1.995\,|E_0|$



4.17 $|E| = 2|E_0|\sin(k_z z)$, $k_z = k\cos 10^\circ$

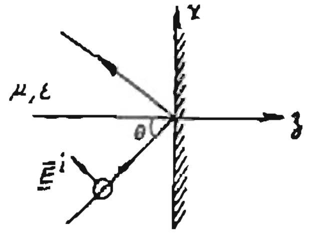
$\therefore |E| = |E_0|\,2\sin(1.969\times 0.8\times\cos 10^\circ) = 1.9996\,|E_0|$



4.18 $\underline{E} = \hat{y}(-2jE_0)\sin k_z z\, e^{-jk_x x}$, $\underline{H} = \frac{\nabla\times\underline{E}}{-j\omega\mu} = \frac{2E_0}{\omega\mu}\left[-\hat{x}k_z\cos k_z z - \hat{z}(jk_x)\sin k_z z\right]e^{-jk_x x}$

$\therefore\ \underline{H}\big|_{z=0} = -\hat{x}\,2E_0\frac{k_z}{\omega\mu}e^{-jk_x x} = -\hat{x}\frac{2E_0}{\eta}\cos\theta\, e^{-jkx\sin\theta}$

4.19



Perfect conductor : $R_{II} = 1$, $T_{II} = 0$

$k_x = k\sin\theta$, $k_z = k\cos\theta$

$\underline{H}^i = \hat{y}H_0 e^{-jkx\sin\theta - jkz\cos\theta}$

$\underline{E}^i = (\hat{x}\cos\theta - \hat{z}\sin\theta)(H_0\eta)e^{-jkx\sin\theta - jkz\cos\theta}$, $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$\underline{H}^r = \hat{y}H_0 e^{-jkx\sin\theta + jkz\cos\theta}$

$\underline{E}^r = -(\hat{x}\cos\theta + \hat{z}\sin\theta)(H_0\eta)e^{-jkx\sin\theta + jkz\cos\theta}$

4.20 $\bar{k}_i = -\hat{x}k\cos\theta - \hat{y}k\sin\theta$, $\bar{k}_u = -\hat{x}k\cos\theta + \hat{y}k\sin\theta$

$\bar{k}_d = \hat{x}\cos\theta - \hat{y}k\sin\theta$, $\bar{k}_r = \hat{x}k\cos\theta + \hat{y}k\sin\theta$; $R_I = -1$

$\underline{E}^i = \hat{z}E_0 e^{jkx\cos\theta + jky\sin\theta}$, $\underline{E}^u = (-1)\hat{z}E_0 e^{jkx\cos\theta - jky\sin\theta}$

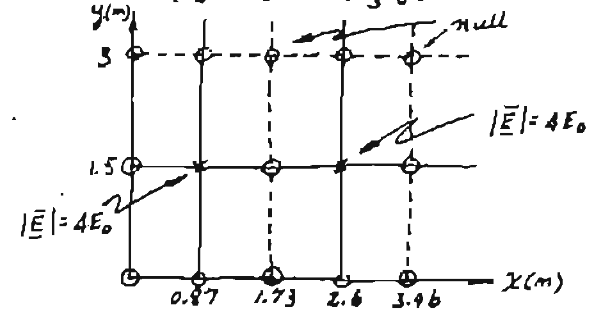
$\underline{E}^d = (-1)\hat{z}E_0 e^{-jkx\cos\theta + jky\sin\theta}$, $\underline{E}^r = \hat{z}E_0 e^{-jkx\cos\theta - jky\sin\theta}$

$\underline{E} = \underline{E}^i + \underline{E}^d + \underline{E}^u + \underline{E}^r = \hat{z}E_0\left[e^{jkx\cos\theta}(e^{jky\sin\theta} - e^{-jky\sin\theta}) - e^{-jkx\cos\theta}(e^{jky\sin\theta} - e^{-jky\sin\theta})\right]$

$= \hat{z}E_0(2j)\sin(ky\sin\theta)(2j)\sin(kx\cos\theta) = \hat{z}(-4E_0)\sin(kx\cos\theta)\sin(ky\sin\theta)$

4.21 $f = 100\,MHz \Rightarrow k = \frac{2\pi\times10^8}{3\times10^8} = \frac{2}{3}\pi$, $\theta = 30° \Rightarrow \sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

$\therefore\ \underline{E} = \hat{z}(-4E_0)\sin(\frac{\sqrt{3}}{3}\pi x)\sin(\frac{\pi}{3}y)$



antenna location at
$\begin{cases} x = .87m \\ y = 1.5m \end{cases}$

$Gain = 20\log_{10}\left|\frac{4E_0}{E_0}\right| = 12.04\,dB$

4.22 (i) $R_I = \frac{k_1 - k_2}{k_1 + k_2} = \frac{1-2}{1+2} = -\frac{1}{3}$

$|E_{y\,total}|$ at $z=0$: $1 + R_I = 2/3$

$|E_{y\,total}|$ maximum : $1 + |R_I| = 4/3$

$|E_{y\,total}|$ minimum : $1 - |R_I| = 2/3$

Standing wave pattern : left column, 4th row

4.22 (continued)

(ii) $R_\perp = 1/3$ pattern: left column, third row

(iii) total reflection, $|R| = 1$, medium 2 E field must have exponential decay characteristics, medium 1 E field will have maximum value equal to 2 and minimum zero. Pattern: right column, first row.

(iv) total transmission, no reflection, $|E_x|$ field is a constant in both medium 1 and medium 2. Pattern: right column, third row.

(v) total transmission, no reflection, $|E_z|$ field is constant in medium 1 and is equal to another constant in medium 2. The ratio should be $|E_{z1}| / |E_{z2}| = \epsilon_2/\epsilon_1 < 1$. Pattern: right column, fourth row.

(vi) No E field in medium 2, $|E_{y\,total}| = 0$ at $z = 0$.
Pattern: left column, first row.

(vii) No H field in medium 2. $|H_{y\,total}|\,\eta$ is maximum at $z = 0$
Pattern: left column, fifth row.

4.23 $\bar{B} = \hat{y}\,\dfrac{2\mu E_0}{\eta_0}\cos(\omega t - kz)$

$\bar{F}_m = q\bar{v}\times\bar{B} = \hat{z}\,(2\mu d q^2 E_0^2/\eta_0)\cos^2(\omega t - kz)$

[this is two times the force in equation (3.40)]

Following the derivation given in the text, one obtains

$\bar{p} = \frac{2}{c}\langle \bar{S}\rangle$.

Using (5.45), we obtain

$$\nabla \times \mathbf{A} = \hat{\phi}\frac{\partial A_z}{\partial z} = \hat{\phi}\frac{-jk}{\rho}e^{-jkz}$$

Example 5.8

Calculate the total time-average electromagnetic power transmitted along a coaxial line when the fields are given by (5.48).

Solution: The time-average Poynting vector $\langle \mathbf{S} \rangle$ is given by

$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] = \frac{1}{2}\mathrm{Re}\left\{\frac{V_0}{\rho}e^{-jkz}\hat{\rho} \times \hat{\phi}\frac{V_0}{\eta\rho}e^{jkz}\right\} = \hat{z}\frac{V_0^2}{2\eta\rho^2}$$

Therefore $P = (\pi V_0^2/\eta)/\ln(b/a)$.

Problems

5.1 Show that the complete fields of the TE wave in a parallel-plate waveguide are given as follows:

$$E_y = E_0 \sin k_x x\, e^{-jk_z z}$$

$$H_x = \frac{-k_z E_0}{\omega\mu}\sin k_x x\, e^{-jk_z z}$$

$$H_z = \frac{jk_x E_0}{\omega\mu}\cos k_x x\, e^{-jk_z z}$$

$$E_x = 0;\ E_z = 0;\ H_y = 0$$

5.2 Find the complete fields of the TM wave in a parallel-plate waveguide.

5.3 What is the lowest frequency of an electromagnetic wave that can be propagated in the TE mode in the earth-ionosphere waveguide? Model the latter as two perfectly conducting parallel plates separated by 80 km.

5.4 Find the surface-charge density ρ_s on the upper and the lower plates of a parallel-plate waveguide for (a) the TE_m mode, and (b) the TM_m mode.

5.5 Find the mathematical expressions for a TEM wave in a parallel-plate waveguide that propagates in the $-\hat{z}$ direction (see Figure 5.1). Sketch the parallel-plate waveguide, and indicate the directions of **E**, **H**, and $\mathbf{J}_s$.

5.6 A microstrip line has the dimensions $a = 0.15$ cm and $w = 0.71$ cm, and the permittivity of the substrate is $\epsilon = 2.5\,\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$. Estimate the time-average power that is transmitted by the line when $|\mathbf{E}| = 10^4$ V/m.

5.7 The breakdown voltage of the dielectric substrate used in the stripline described in Problem 5.6 is 2×10^7 V/m. Use a safety factor of 10 so that $|\mathbf{E}|$ is less than 2×10^6 V/m everywhere in the line. Find the maximum time-average power that the stripline can transmit. Neglect the ohmic loss.

5.8 With the fields in a rectangular waveguide, find the surface current $\mathbf{J}_s$ on the top $(y = b)$ of the waveguide. If we want to cut a slot along z, where should the slot be cut in order to minimize the disturbance it will cause? Assume that only the TE_{10} mode exists in the waveguide.

5.9 Show that, if the wavelength of an electromagnetic wave in an unbounded medium characterized by μ and ϵ is greater than $2a$, then this wave cannot propagate in the rectangular waveguide (shown in Figure 5.8) with the dielectric inside the waveguide also characterized by μ and ϵ.

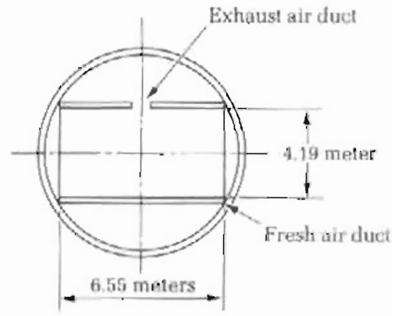


Figure P5.10

5.10 An AM radio in an automobile cannot receive any signal when the car is inside a tunnel. Why? Let us assume that the tunnel is the Lincoln Tunnel, which was built in 1939 under the Hudson River in New York. Figure P5.10 shows a cross section of the Lincoln Tunnel.*

5.11 Find the frequency ranges for TE_{10} mode operation for those rectangular waveguides listed in Table 1.

5.12 Design an air-filled rectangular waveguide to be used for transmission of electromagnetic power at 2.45 GHz. This frequency should be at the middle of the operating frequency band. The design should also allow maximum power transfer without sacrificing the operating frequency bandwidth. Find the maximum power the waveguide can transmit. Use a safety factor of 10. Neglect ohmic loss. The breakdown E in air is assumed to be 2×10^6 V/m.

5.13 Repeat Problem 5.12, but assume that a dielectric material is used to fill the waveguide. The material is characterized by $\epsilon = 2.50\epsilon_0$, $\mu = \mu_0$, and $\sigma = 0$. The breakdown E field in the dielectric is 10^7 V/m.

5.14 Consider the size of a rectangular waveguide to explain why it is not used to transmit electromagnetic waves in the VHF range. (Take $f = 100$ MHz.)

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*G. E. Sandstrom, *Tunnels*, New York: Holt, Rinehart & Winston, 1963, p. 242.

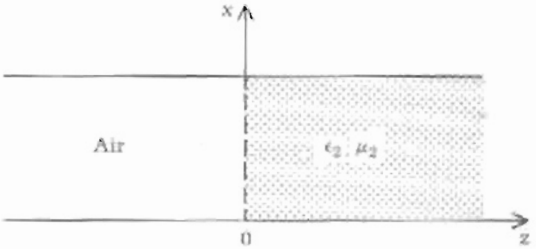


Figure P5.16

5.15 The electromagnetic fields associated with the TE_{10} mode propagating in the $\hat{z}$ direction are given by (5.23). Find the electromagnetic fields associated with the TE_{10} mode propagating in the $-\hat{z}$ direction, with maximum electric field equal to E_1.

5.16 Consider a rectangular waveguide shown in Figure P5.16. For the region $z < 0$, the medium is air and for $z > 0$ the medium is characterized by ϵ_2 and μ_2. A TE_{10} mode with maximum E-field equal to E_0 impinges on the boundary from the left. The result is that some power is reflected and some is transmitted. Assume that the reflected wave is also TE_{10}, with maximum E-field equal to E_1, and the transmitted wave is TE_{10} mode with maximum E-field equal to E_2. Find the ratio E_1/E_0 in terms of a, ω, ϵ_0, μ_0, ϵ_2, and μ_2.

5.17 The corner reflector studied in Problem 4.20 requires the solution

$$\mathbf{E} = -\hat{z}4E_0 \sin(kx\cos\theta)\sin(ky\sin\theta)$$

Show that although the coordinates are different this solution is in fact the resonator mode that we studied in Section 5.2. Placing conducting plates at $x = a$ and $y = b$ to form a cavity resonator as shown in Problem 4.20, what are the restrictions on the incident angle θ?

5.18 (a) Find the real-time expression of the fields of the TE_{101} mode in the rectangular cavity shown in Figure 5.9.

(b) Find the total stored electric energy in the cavity as a function of time. Find the corresponding total stored magnetic energy.

(c) Show that energy is stored alternatingly in electric and in magnetic fields, that the maximum stored electric energy is equal to the maximum stored magnetic energy, and that the total stored electromagnetic energy in the cavity is a constant independent of time. Note that these properties are similar to those of the low-frequency LC resonant circuits.

5.19 Find the lowest resonant frequency of the TE_{101} mode in an air-filled rectangular cavity measuring $2 \times 3 \times 5$ cm^3. Note that there are three different choices for designating the z axis and that these result in three different TE_{101} modes.

5.20 Electromagnetic waves in air with wavelengths ranging from 1 to 10 mm are called millimeter waves. Millimeter waves may be guided by dielectric slabs. Consider a dielectric slab with $\epsilon_1 = 10\epsilon_0$ and $\epsilon_2 = \epsilon_0$, as shown in Figure 5.12. What should its thickness be in order that only the TE_0 mode may be excited for frequencies up to 300 GHz?

5.21 Use direct substitution into Maxwell's equations to show that the fields given by (5.48) are solutions of Maxwell's equations in cylindrical coordinates.

5.22 Use the formulas of divergence and curl in cylindrical coordinates to prove that $\nabla \cdot \nabla \times \mathbf{A} = 0$ for any vector A.

5.23 Find the rectangular coordinates of a point P where the cylindrical coordinates are $\rho = 1$, $\phi = 30°$, and $z = 2$.

5.24 Find the cylindrical coordinates of a point Q where the rectangular coordinates are x, y, and z.

5.25 Show that the differential volume in the cylindrical coordinates is $\rho\, d\rho\, d\phi\, dz$.

5.26 To convert a vector expressed in cylindrical components into the same in rectangular components, or vice versa, it is convenient to prepare a table for dot products between unit vectors in these coordinate systems. For example, $\hat{\mathbf{x}} \cdot \hat{\rho} = \cos \phi$, as shown in the following table. Complete the table.

Dot Products Between Cylindrical and Rectangular Unit Vectors

	$\hat{\rho}$	$\hat{\phi}$	$\hat{\mathbf{z}}$
$\hat{\mathbf{x}}$	$\cos \phi$		
$\hat{\mathbf{y}}$			
$\hat{\mathbf{z}}$	0		

5.27 Use the above table to find the rectangular components of the following vector located at $\rho = 2$, $\phi = 30°$, and $z = -3$:

$$A = 8\hat{\rho} + 4\hat{\phi} - 3\,\hat{\mathbf{z}}$$

5.28 What is the maximum time-average power a coaxial line can transmit without causing breakdown? Assume that the coaxial line is air-filled and that the breakdown E of the air is 2×10^6 V/m. Use a safety factor of 10 so that the maximum E field anywhere in the line does not exceed 2×10^5 V/m. The dimension of the line is $2a = 0.411$ cm and $2b = 1.143$ cm. Neglect ohmic loss.

5.29 Consider the coaxial line shown in Figure P5.29. Half of the line ($z < 0$) is filled with air, and half of it ($z > 0$) is filled with a material characterized by ϵ_1 and μ_1. The electromagnetic wave incident from the left has the following fields:

$$\mathbf{E}^i = \hat{\rho}\frac{V_0}{\rho}e^{-jk_0 z}$$

$$\mathbf{H}^i = \hat{\phi}\frac{V_0}{\eta_0 \rho}e^{-jk_0 z}$$

The fields of the reflected wave may be expressed as follows:

$$\mathbf{E}^r = \hat{\rho}\frac{V_0'}{\rho}e^{jk_0 z}$$

$$\mathbf{H}^r = \hat{\phi}\frac{-V_0'}{\eta_0 \rho}e^{jk_0 z}$$

(a) Write down the fields of the transmitted wave in $z > 0$. What wave number k should be used?

(b) Find V_0' and the amplitude of the transmitted fields in terms of V_0, η_1, and η_0 by matching the boundary conditions at $z = 0$. Compare your result with the reflection and transmission coefficients obtained in Chapter 4 for waves reflected from dielectric boundaries.

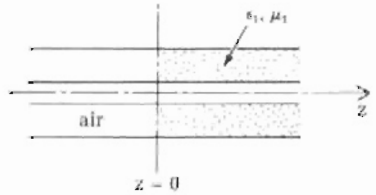
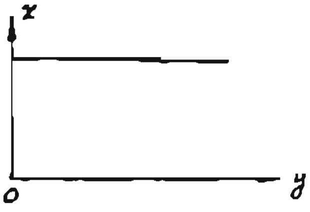


Figure P 5.29

CHAPTER 5



5.1 $\nabla\times\bar{H} = j\omega\varepsilon\bar{E}$ and $\nabla\times\bar{E} = -j\omega\mu\bar{H}$ with $\partial/\partial y = 0$

$$\Rightarrow \left.\begin{aligned} -\frac{\partial}{\partial z}E_y &= -j\omega\mu H_x\\ \frac{\partial}{\partial x}E_y &= -j\omega\mu H_z \\ \frac{\partial}{\partial z}H_x - \frac{\partial}{\partial x}H_z &= j\omega\varepsilon E_y\end{aligned}\right\} \Rightarrow \left(\frac{\partial^2}{\partial^2 x}+\frac{\partial^2}{\partial^2 z}+\omega^2\mu\varepsilon\right)E_y = 0$$

Assume $E_y = A(x)e^{-jk_z z}$, then $\frac{d^2A(x)}{dx^2}+\omega^2\mu\varepsilon-k_z^2=0$

∴ $\frac{d^2A(x)}{dx^2}+k_x^2=0 \Rightarrow A(x)=C_1\cos k_x x + C_2\sin k_x x$, where $k_x^2=\omega^2\mu\varepsilon-k_z^2$

∴ $E_y = C_1\cos k_x x\, e^{-jk_z z} + C_2 \sin k_x x\, e^{-jk_z z}$

$E_y|_{x=0}=0 \Rightarrow C_1=0$; $E_y|_{x=a}=0 \Rightarrow k_x=\frac{m\pi}{a}$, $m=1,2,3,\cdots$

∴ $E_y = E_0\sin k_x x\, e^{-jk_z z}$ (denoted C_2 by E_0)

$\bar{H} = \frac{\nabla\times\bar{E}}{-j\omega\mu} = -\hat{x}\frac{k_z}{\omega\mu}E_0\sin k_x x\, e^{-jk_z z} + \hat{z}\,j\frac{E_0}{\omega\mu}k_x\cos k_x x\, e^{-jk_z z}$

∴ $H_x = -\frac{k_z}{\omega\mu}E_0 \sin k_x x\, e^{-jk_z z}$; $H_z = j\frac{k_x}{\omega\mu}E_0\cos k_x x\, e^{-jk_z z}$; and $E_x = E_z = H_y = 0$

5.2 Assume $\partial/\partial y=0$ and $H_y = H(x)e^{-jk_z z}$, then $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\omega^2\mu\varepsilon\right)H_y = \frac{d^2H(x)}{dx^2}+(\omega^2\mu\varepsilon-k_z^2)=0$

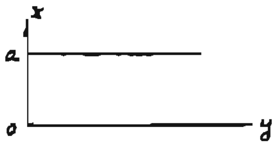
$H(x)=A\cos k_x x + B\sin k_x x$, where $k_x^2=\omega^2\mu\varepsilon-k_z^2$

∴ $H_y = A\cos k_x x\, e^{-jk_z z}+B\sin k_x x\, e^{-jk_z z}$ and $\bar{E}=\frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \hat{x}\left(\frac{k_z}{\omega\varepsilon}\right)(A\cos k_x x + B\sin k_x x)e^{-jk_z z}$
$+ \hat{z}\left(\frac{k_x}{j\omega\varepsilon}\right)(-A\sin k_x x + B\cos k_x x)e^{-jk_z z}$

$E_z|_{x=0}=0 \Rightarrow B=0$

$$\therefore \begin{cases} E_x = \frac{k_z}{\omega\varepsilon}H_0\cos k_x x\, e^{-jk_z z} \quad \text{(denoted } A \text{ by } H_0)\\ E_z = \frac{jk_x}{\omega\varepsilon}H_0\sin k_x x\, e^{-jk_z z} \\ H_y = H_0\cos k_x x\, e^{-jk_z z} \\ H_x = H_z = E_y = 0 \end{cases}$$

5.3 $f_c = \frac{1}{2a\sqrt{\mu\varepsilon}} = \frac{3\times10^8}{2\times80\times10^3} = 1.875$ KHz



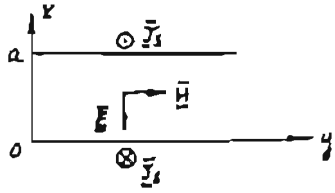
5.4 (a) TE_m: $E_y = E_0\sin\left(\frac{m\pi}{a}x\right)e^{-jk_z z}$

$E_y|_{x=0} = E_y|_{x=a}=0 \Rightarrow \rho_s = 0$ on both plates

(b) TM_m: $E_x = \frac{k_z}{\omega\varepsilon}H_0\cos\left(\frac{m\pi}{a}x\right)e^{-jk_z z}$ (From Problem 5.2)

at $x=0$, $\rho_s = \hat{x}\cdot\varepsilon\bar{E}|_{x=0} = \frac{k_z}{\omega}H_0e^{-jk_z z}$

at $x=a$, $\rho_s = -\hat{x}\cdot\varepsilon\bar{E}|_{x=a} = -\frac{k_z}{\omega}H_0(-1)^m e^{-jk_z z} = (-1)^{m+1}\frac{k_z}{\omega}H_0 e^{-jk_z z}$, $m=0,1,2,\cdots$



5.5 TEM in parallel plates = TM_0

propagating in $-z$ direction : $\bar{H}=\hat{y}H_0e^{jkz}$, $k=\omega\sqrt{\mu\varepsilon}$

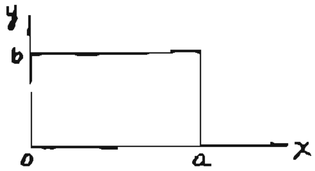
$\bar{E} = \frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \frac{H_0}{j\omega\varepsilon}(-jk\hat{x})e^{jkz} = -\hat{x}H_0\eta e^{jkz}$

at $x=0$, $\bar{J}_s = \hat{x}\times\bar{H}|_{x=0} = \hat{z}H_0e^{jkz}$

at $x=a$, $\bar{J}_s = -\hat{x}\times\bar{H}|_{x=a} = -\hat{z}H_0e^{jkz}$

5.6 $P = \frac12|E|^2\frac{wa}{\eta} = \frac{(10^4)^2\times7.1\times10^{-3}\times1.5\times10^{-3}}{2\times120\pi\times(1/\sqrt{2.5})} = 2.233$ W

5.7 $|E| = 2\times10^6$ and $P = \frac{(2\times10^6)^2\times7.1\times10^{-3}\times1.5\times10^{-3}}{2\times120\pi\times(1/\sqrt{2.5})} = 89.33$ kW



5.8 From equation (5.166), with $m=0$,

$\bar{J}_s(y=b) = -\hat{y}\times\bar{H}|_{y=b} = -\hat{z}\left(\frac{k_zE_0}{\omega\mu}\right)\sin\left(\frac{\pi x}{a}\right)e^{-jk_z z} - \hat{x}\left(\frac{jE_0\pi}{\omega\mu a}\right)\cos\left(\frac{\pi x}{a}\right)e^{-jk_z z}$

at $x=\frac{a}{2}$, $\bar{J}_s(y=b) = -\hat{z}\frac{k_zE_0}{\omega\mu}e^{-jk_z z}$, $\hat{z}$ component of current only

∴ The slot should cut along z at the middle (i.e. at $x=a/2$).

5.9 $f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2} \Rightarrow \lambda_c = \frac{2}{\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}}$

Assume $b<a$, then the longest λ_c is at $m=1$ and $n=0$; i.e. $\lambda_c = 2a$

$\therefore$ wave with $\lambda > \lambda_c = 2a$ cannot propagate in the rectangular waveguide.

5.10 $(f_c)_{TE_{10}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\,2a} = \frac{3\times10^8}{2\times6.55} = 22.9$ MHz, much higher than AM frequency (≈ 1 MHz).

5.11 $(f_c) = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2} \Rightarrow (f_c)_{10} = \frac{1}{2a\sqrt{\mu\varepsilon}},\ (f_c)_{01} = \frac{1}{2b\sqrt{\mu\varepsilon}},\ (f_c)_{20} = \frac{1}{a\sqrt{\mu\varepsilon}}$

For $2b > a$, then $(f_c)_{10} \sim (f_c)_{01}$ is the operating frequency range

For $2b < a$, then $(f_c)_{10} \sim (f_c)_{20}$ is the operating frequency range

For 2.85×1.262, $a>2b$; frequency range is from $\frac{3\times10^8}{2\times2.85\times10^{-2}}$ (= 5.26 GHz) to $\frac{3\times10^8}{2.85\times10^{-2}}$ (= 10.53 GHz).

For 0.711×0.355, $a>2b$; frequency range is from $\frac{3\times10^8}{2\times0.711\times10^{-2}}$ (= 21.1 GHz) to $\frac{3\times10^8}{0.711\times10^{-2}}$ (= 42.2 GHz).

5.12 Safety factor of 10 $\Rightarrow |\vec{E}|_{max} = 2\times10^6\times10 = 2\times10^5$ V/m

$(f_c)_{10} = \frac{3\times10^8}{2a}$; for maximum power transfer, $b=\frac{a}{2}$, $(f_c)_{20} = (f_c)_{01} = \frac{3\times10^8}{a}$

$2.45\text{ GHz} = \frac{1}{2}[(f_c)_{10}+(f_c)_{01}] = \frac{1}{2}(\frac{1}{2a}+\frac{1}{a})\times3\times10^8 = \frac{9}{4a}\times10^8 \Rightarrow a = 9.18\text{ cm}$ and $b = \frac{a}{2} = 4.59$ cm

$\lambda = \frac{3\times10^8}{2.45\times10^9} = 12.24$ cm, $Z_{TE} = \frac{\omega\mu}{k_z} = \frac{\omega\mu}{\sqrt{k^2-(\pi/a)^2}} = \frac{\eta}{\sqrt{1-(\lambda/2a)^2}} = \frac{377}{\sqrt{1-(12.24/18.37)^2}} = 506\ \Omega$

$P = \frac{|E|^2_{max}}{4Z_{TE}}(ab) = \frac{(2\times10^5)^2\times9.18\times10^{-2}\times4.59\times10^{-2}}{4\times506} = 83.3$ KW

5.13 $(f_c)_{10} = \frac{3\times10^8}{2a\sqrt{2.5}},\ (f_c)_{01} = \frac{3\times10^8}{a\sqrt{2.5}}$; $\lambda = \frac{3\times10^8}{\sqrt{2.5}\times2.45\times10^9} = 7.74$ cm

$2.45\text{ GHz} = \frac{1}{2}[(f_c)_{10}+(f_c)_{01}] = \frac{1}{2\sqrt{2.5}}(\frac{1}{2a}+\frac{1}{a})\times3\times10^8 = \frac{9}{4\sqrt{2.5}\,a}\times10^8 \Rightarrow a = 5.81$ cm and $b=\frac{a}{2} = 2.90$ cm

$Z_{TE} = \frac{\eta}{\sqrt{1-(\lambda/2a)^2}} = \frac{377}{\sqrt{2.5}\sqrt{1-(7.74/11.62)^2}} = 320\ \Omega$

Safety factor 10 $\Rightarrow |E|_{max} = 10^6$ $\therefore P = \frac{|E|^2_{max}}{4Z_{TE}}(ab) = \frac{(10^6)^2\times5.81\times10^{-2}\times2.9\times10^{-2}}{4\times320} = 1.32$ MW

5.14 $(f_c)_{10} = \frac{1}{2a}\frac{1}{\sqrt{\mu\varepsilon}} = \frac{3\times10^8}{2a}$; $f = 100\text{ MHz} = 10^8 > (f_c)_{10} = \frac{3\times10^8}{2a} \Rightarrow a > 1.5$ m, physically too large!

5.15 $E_y = E_1\sin\frac{\pi x}{a}e^{jk_z z}$, $H_x = -\frac{k_z}{\omega\mu}E_1\sin\frac{\pi x}{a}e^{jk_z z}$, $H_z = j\frac{\pi/a}{\omega\mu}E_1\cos\frac{\pi x}{a}e^{jk_z z}$

where $k_z = \sqrt{\omega^2\mu\varepsilon - (\frac{\pi}{a})^2}$.

5.16 $E_{y1} = (E_0 e^{-jk_{1z}z} + E_1 e^{jk_{1z}z})\sin\frac{\pi x}{a}$; $E_{y2} = E_2\sin\frac{\pi x}{a}e^{-jk_{2z}z}$

$E_{y1} = E_{y2}$ at $z=0 \longrightarrow \boxed{E_0 + E_1 = E_2}$

$H_{x1} = (E_0e^{-jk_{1z}z} - E_1e^{jk_{1z}z})\frac{-k_{1z}}{\omega\mu_0}\sin\frac{\pi x}{a}$; $H_{x2} = \frac{-k_{2z}}{\omega\mu_2}E_2\sin\frac{\pi x}{a}e^{jk_{2z}z}$

$H_{x1} = H_{x2}$ at $z=0 \longrightarrow \boxed{(E_0 - E_1)\frac{k_{1z}}{\mu_0} = E_2\frac{k_{2z}}{\mu_2}}$

solve these two equations:

$$\frac{E_1}{E_0} = \frac{\mu_2k_{1z} - \mu_0k_{2z}}{\mu_2k_{1z} + \mu_0k_{2z}}$$

where $k_{1z} = \sqrt{\omega^2\mu_0\varepsilon_0 - (\frac{\pi}{a})^2}$, $k_{2z} = \sqrt{\omega^2\mu_2\varepsilon_2 - (\frac{\pi}{a})^2}$.

5.17 $\bar{E}=0$ at $x=a \Rightarrow \sin(ka\cos\theta)=0 \Rightarrow ka\cos\theta = m\pi,\ m=1,2,3,\cdots$

$\bar{E}=0$ at $y=b \Rightarrow \sin(kb\sin\theta)=0 \Rightarrow kb\sin\theta = n\pi,\ n=1,2,3,\cdots$

$\therefore \tan\theta = \frac{na}{mb}$; $m=1,2,3,\cdots$, $n=1,2,3,\cdots$

5.18 (a) $E_y = E_{11}\sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\cos\omega t$

$H_x = \frac{E_{11}}{\omega\mu}\frac{\pi}{d}\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}\sin\omega t$

$H_z = -\frac{E_{11}}{\omega\mu}\frac{\pi}{a}\cos\frac{\pi x}{a}\sin\frac{\pi z}{d}\sin\omega t$

(b) $U_E = \frac{\varepsilon}{2}E_{11}^2\cos^2\omega t\int_0^a\int_0^b\int_0^d \sin^2(\frac{\pi x}{a})\sin^2(\frac{\pi z}{d})\,dx\,dy\,dz = \frac{\varepsilon}{8}E_{11}^2\cos^2\omega t\,(abd)$

$U_H = \frac{\mu}{2}E_{11}^2(\frac{\pi}{\omega\mu})^2\int_0^a\int_0^b\int_0^d\{\frac{1}{d^2}\sin^2(\frac{\pi x}{a})\cos^2(\frac{\pi z}{d})\sin^2\omega t + \frac{1}{a^2}\cos^2(\frac{\pi x}{a})\sin^2(\frac{\pi z}{d})\sin^2\omega t\}\,dx\,dy\,dz$

$= \frac{E_{11}^2\pi^2}{8\omega^2\mu}\sin^2\omega t\,(abd)(\frac{1}{a^2}+\frac{1}{d^2})$

(c) $U_E^{max} = \frac{\varepsilon}{8}E_{11}^2(abd)$; $U_H^{max} = \frac{\pi^2}{8\omega^2\mu}E_{11}^2(abd)(\frac{1}{a^2}+\frac{1}{d^2})$

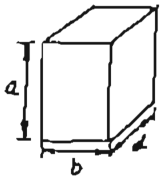
At resonance, $f_{11} = \frac{1}{2\sqrt{\mu\varepsilon}}(\frac{1}{a^2}+\frac{1}{d^2})^{1/2} \Rightarrow \omega^2 = (2\pi f_{11})^2 = \frac{\pi^2}{\mu\varepsilon}(\frac{1}{a^2}+\frac{1}{d^2})$,

therefore $U_H^{max} = \frac{\pi^2}{8\mu}E_{11}^2(abd)(\frac{1}{a^2}+\frac{1}{d^2})\times\frac{\mu\varepsilon}{\pi^2}(\frac{1}{a^2}+\frac{1}{d^2})^{-1} = \frac{\varepsilon}{8}E_{11}^2(abd) = U_E^{max}$

At resonance, $\omega^2 = \frac{\pi^2}{\mu\varepsilon}(\frac{1}{a^2}+\frac{1}{d^2}) \Rightarrow U_H = \frac{\varepsilon}{8}E_{11}^2(abd)\sin^2\omega t$,

therefore $U^{total} = U_H + U_E = \frac{\varepsilon}{8}E_{11}^2(abd)[\sin^2\omega t + \cos^2\omega t] = \frac{\varepsilon}{8}E_{11}^2(abd) =$ constant

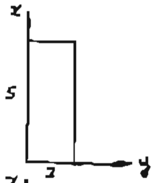
5.19



cavity size = $2\times3\times5\ cm^3$

$(f_c)_{101} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\frac{1}{a^2}+\frac{1}{d^2}}$

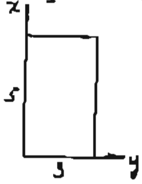
(i) Choose $a = 5cm$



$d = 3cm$

$f_c = \frac{3\times10^8}{2}\sqrt{(\frac{1}{0.05})^2+(\frac{1}{0.03})^2}$

$= 5.83\ GHz$

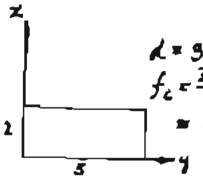


$d = 2cm$

$f_c = \frac{3\times10^8}{2}\sqrt{(\frac{1}{0.05})^2+(\frac{1}{0.02})^2}$

$= 8.08\ GHz$

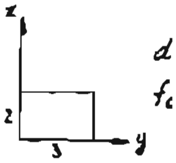
(ii) Choose $a = 2cm$



$d = 3cm$

$f_c = \frac{3\times10^8}{2}\sqrt{(\frac{1}{0.02})^2+(\frac{1}{0.03})^2}$

$= 9.01\ GHz$

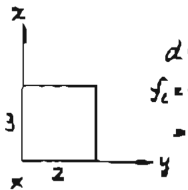


$d = 5cm$

$f_c = \frac{3\times10^8}{2}\sqrt{(\frac{1}{0.02})^2+(\frac{1}{0.05})^2}$

$= 8.08\ GHz$

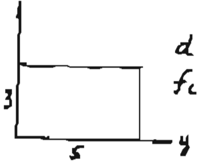
(iii) Choose $a = 3cm$



$d = 5cm$

$f_c = \frac{3\times10^8}{2}\sqrt{(\frac{1}{0.03})^2+(\frac{1}{0.05})^2}$

$= 5.83\ GHz$



$d = 2cm$

$f_c = \frac{3\times10^8}{2}\sqrt{(\frac{1}{0.03})^2+(\frac{1}{0.02})^2}$

$= 9.01\ GHz$

5.20 $f_c = \frac{n}{2d\sqrt{\mu_0(\epsilon_1-\epsilon_0)}} \Rightarrow 2d = \frac{3\times10^8}{300\times10^9\times\sqrt{10-1}} = 0.33\text{ mm}$, for $n=1$

5.21 $\underline{E} = \hat{\rho}\frac{V_0}{\rho}e^{-jkz}$

$$\nabla\times\underline{E} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ \frac{V_0}{\rho}e^{-jkz} & 0 & 0 \end{vmatrix} = \hat{\phi}\frac{1}{\rho}\cdot\rho\cdot\frac{V_0}{\rho}(-jk)e^{-jkz} = -jk\frac{V_0}{\rho}e^{-jkz}\hat{\phi} = -j\omega\mu\sqrt{\frac{\epsilon}{\mu}}\frac{V_0}{\rho}e^{-jkz}\hat{\phi}$$

$$= -j\omega\mu\left[\hat{\phi}\frac{V_0}{\eta\rho}e^{-jkz}\right] = -j\omega\mu\underline{H}$$

$$\Rightarrow \underline{H} = \hat{\phi}\frac{V_0}{\eta\rho}e^{-jkz}$$

5.22 $\nabla\times\bar{A} = \hat{\rho}\left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right] + \hat{\phi}\left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right] + \hat{z}\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]$

$$\nabla\cdot\nabla\times\bar{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\frac{\partial A_z}{\partial\phi} - \rho\frac{\partial A_\phi}{\partial z}\right] + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right] + \frac{1}{\rho}\frac{\partial}{\partial z}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]$$

$$= \frac{1}{\rho}\frac{\partial^2 A_z}{\partial\rho\partial\phi} - \frac{\partial^2 A_\phi}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 A_\rho}{\partial\phi\partial z} - \frac{1}{\rho}\frac{\partial^2 A_z}{\partial\phi\partial\rho} + \frac{\partial^2 A_\phi}{\partial z\partial\rho} + \cancel{\frac{1}{\rho}\frac{\partial A_\phi}{\partial z}} - \frac{1}{\rho}\frac{\partial^2 A_\rho}{\partial z\partial\phi} - \cancel{\frac{1}{\rho}\frac{\partial A_\phi}{\partial z}} = 0$$

5.23 $x = \cos 30° = \sqrt{3}/2$; $y = \sin 30° = 1/2$; $z = 2$

5.24 $\rho = \sqrt{x^2+y^2}$; $\phi = \tan^{-1}(y/x)$; $z = z$

5.25 $dv = d\rho\cdot(\rho d\phi)\cdot dz = \rho\, d\rho\, d\phi\, dz$

5.26

$\cdot$	$\hat{\rho}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}$	$\sin\phi$	$\cos\phi$	0
$\hat{z}$	0	0	1

5.27 $\bar{A} = 8\hat{\rho} + 4\hat{\phi} - 3\hat{z}$; $A_x = \hat{x}\cdot\bar{A} = 8\cos\phi - 4\sin\phi$, $A_y = \hat{y}\cdot\bar{A} = 8\sin\phi + 4\cos\phi$, $A_z = \hat{z}\cdot\bar{A} = -3$

$\bar{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z = \hat{x}(8\cos\phi - 4\sin\phi) + \hat{y}(8\sin\phi + 4\cos\phi) - \hat{z}3$

At $\phi = 30°$, $\bar{A} = \hat{x}(4\sqrt{3} - 2) + \hat{y}(4 + 2\sqrt{3}) - \hat{z}3 = \hat{x}4.93 + \hat{y}7.46 - \hat{z}3$

5.28 $|\underline{E}|^{max} = 2\times10^5 = \frac{V_0}{a} \Rightarrow V_0 = 2\times10^5\times(0.00411/2) = 411$

$$P = \int_0^{2\pi}\int_a^b \frac{V_0^2}{2\eta}\frac{1}{\rho^2}\rho\, d\rho\, d\phi = \frac{V_0^2}{2\eta}\cdot 2\pi(\ln\rho)\Big|_a^b = \frac{\pi V_0^2}{\eta}\ln(b/a) = \frac{\pi\times(411)^2\times\ln(1.143/0.411)}{120\pi} = 1.44\text{ kW}$$

5.29 (a) $\underline{E}^t = \hat{\rho}\frac{V_1}{\rho}e^{-jk_1 z}$ and $\underline{H}^t = \hat{\phi}\frac{V_1}{\eta_1\rho}e^{-jk_1 z}$, with $k_1 = \omega\sqrt{\mu_1\epsilon_1}$

(b) At $z=0$, $\underline{E}^i + \underline{E}^r = \underline{E}^t \Rightarrow V_0 + V_0' = V_1$ and $\underline{H}^i + \underline{H}^r = \underline{H}^t \Rightarrow \frac{V_0}{\eta_0} - \frac{V_0'}{\eta_0} = \frac{V_1}{\eta_1} \Rightarrow V_0 - V_0' = \frac{\eta_0}{\eta_1}V_1$

$\therefore V_1 = \frac{2\eta_1}{\eta_0+\eta_1}V_0$ and $V_0' = V_1 - V_0 = \frac{\eta_1-\eta_0}{\eta_1+\eta_0}V_0$

From chapter 4, $V_0'/V_0 = R = \frac{\eta_1-\eta_0}{\eta_1+\eta_0}$ and $V_1/V_0 = T = \frac{2\eta_1}{\eta_0+\eta_1}$

We can find the reflected wave by carrying out an analysis similar to the one for a transmission line with a capacitor. The result is as follows:

$$V_- = V_L - V_+ = V_0\left(e^{-Z_0(t-T)/L} - \tfrac{1}{2}\right) \tag{6.58}$$

Figure 6.39b shows the voltage $V(z)$ on the line during the time period $T < t < 2T$.

Problems

6.1. What is the voltage in the stripline discussed in Example 6.1 when the time-average power being transmitted is 10 kW?

6.2. Consider the coaxial line discussed in Example 6.3. Calculate the maximum time-average power that may be transmitted in the line. Use the breakdown $E = 2 \times 10^7$ V/m and a safety factor of 10.

6.3. Two coaxial lines have equal characteristic impedances: 50 Ω. Both are air-filled. The first line has a power capacity of 1 MW, and the second line's capacity is 1 kW. Find the ratios a_1/a_2 and b_1/b_2. Consider only the breakdown voltage.

6.4. Use (6. 1b) and the boundary condition (4.3) to obtain the surface-current density $\mathbf{J}_s$ on the lower plate of the parallel-plate waveguide. Then calculate the total current on the lower plate. Compare the current with the definition of $I(z)$ given by (6.3b).

6.5. Find the surface-current density $\mathbf{J}_s$ on the inner conductor of a coaxial line. Then calculate the total current on it. Compare the total current with $I(z)$ defined for the coaxial line.

6.6. A transmission line is short-circuited ($Z_L = 0$)

- (a) Find the expressions for $|V(z)|$ and $|I(z)|$ as a function of kz, Z_0, and V_-.
- (b) Sketch $|V(z)|$ and $|I(z)|$
- (c) Find VSWR on the line.

6.7. Repeat Problem 6.6 for a transmission line with an open circuit at the load ($Z_L = \infty$).

6.8. Repeat Problem 6.6 for a transmission line with a matched load ($Z_L = Z_0$).

6.9. A transmission line is terminated with a normalized load of $0.8 + j1.0$. Calculate (a) the VSWR, (b) the position of the voltage minimum, and (c) the percentage of the incident power that is reflected by the load. Sketch $|V(z)|$ as a function of z/λ.

6.10. Solve the problem discussed in Example 6.6 by using the Smith chart. Find the position of a shunt susceptance that can tune the line to have a perfect match. Determine the value (in mhos) of the shunt susceptance.

6.11. For an open-circuited 50 Ω transmission line of length ℓ, the input impedance at the other end is $j33$ Ω. Find the length ℓ (in λ).

6.12. Repeat Problem 6.11 when the line is short-circuited at one end.

6.13. For the first waveguide in Table 1 of Section 5.2, design an iris that will give a $j1.57$ admittance at $f = 8$ GHz.

6.14. From the Smith chart, find Γ_L for the following Z_{Ln}: (a) $1 + j2$, (b) ∞, (c) 0, and (d) $0.55 - j0.38$.

6.15. Use the Smith chart to find Z_{Ln} from the following Γ_L (a) $0.6\, e^{j45^\circ}$, (b) -0.3, and (c) 0.

6.16. For a load impedance of $0.4 - j0.5$, find the location of the first voltage minimum and the first voltage maximum at the load end.

6.17. From the Smith chart, find the admittances for the following impedances: (a) $Z_n = 0.3 - j0.6$ and (b) $Z_w = 5 + j3$.

6.18. A transmission line is terminated with a normalized impedance $Z_{Ln} = 2 + j2$, as shown in Figure 6.19a. The incident $V_+ = 1.0$, and the characteristic impedance of the line is 1.0. Show that $V_{max} = I_{max} = 1.62$, $|V(0)| = 1.55$, $|V(-0.219\lambda)| = 0.78$, $V_{min} = I_{min} = 0.38$, $|I(0)| = 0.55$, and $|I(-0.219\lambda)| = 1.45$.

6.19. A shunt admittance of $Y_{Ln} = -j1.57$ is added to the transmission line that is terminated by a load $Z_{Ln} = 2 + j2$, as shown in Figure 6.19b. The position of the shunt is $0.219\lambda_g$ from the load, so that the line is perfectly matched. Let $V_+ = 1.0$ and $Z_0 = 1.0$ and show that $V_{max} = 2.08$, $|V(0)| = 2.00$, $I_{max} = 1.86$, $|I(0)| = 0.71$, and $I_{min} = 0.49$.

6.20. In Example 6.8, find another set of solutions of ℓ_1 and ℓ_2 (in centimeters).

6.21. For the solution found in Example 6.8, how much total time-average power can be fed to the array without causing breakdown in the dielectric? Use the value 181,000 V/cm as the breakdown strength of the dielectric, use a safety factor 10, and let $a = 2$ mm. **Hint:** consider the standing wave on the stub tuners as well as on the transmission lines.

6.22. For the circuit shown in Figure 6.25a, let $Z_0 = 50\ \Omega$, $R_L = 70\ \Omega$, $R_g = 50\ \Omega$, $\ell = 2$ m, $v = 10^8$ m/s, $\Delta = 10^{-9}$ s, and $V_0 = 1$. Plot the voltage and current at $z = \ell/2$ as a function of time.

6.23. Calculate the percentage of energy generated by the pulse generator that is absorbed by the load in the circuit of Problem 6.22.

6.24. For a four-digit code system, design a D-A converter similar to that discussed in Section 6.5 using the transmission line shown in Figure 6.27a. Specify the value of R, the location of the sampler, and the time that a sample should be taken.

6.25. In plotting Figure 6.32, it is implicitly assumed that $R_L > Z_0$ and that $R_g > Z_0$, so that both Γ_L and Γ_g are positive numbers. Sketch a similar diagram for the case in which $R_L = 0.5Z_0$ and $R_g = 0.5Z_0$.

6.26. Draw the voltage and the current reflection diagrams for the transmission line which is short-circuited as shown in Figure P6.26. Plot V and I as functions of time at $z = \ell/2$.

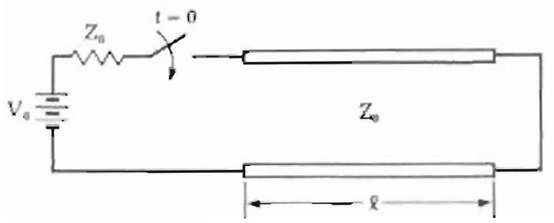


Figure P6.26

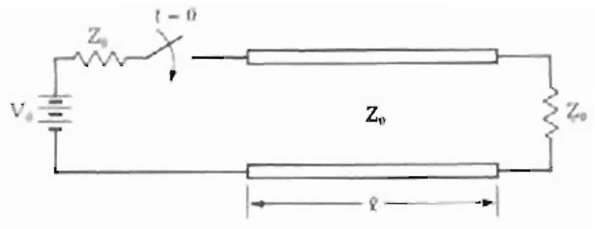


Figure P6.27

6.27. Draw the voltage and the current reflection diagrams for the transmission line that is perfectly matched, as shown in Figure P6.27. Plot V and I as functions of time at $z = \ell/2$.

6.28. Refer to Figure 6.31, and let $R_g = 2Z_0$ and $R_L = 0.5Z_0$. Draw the voltage reflection diagram for $0 < t < 6T$, and plot V at $z = 3\ell/4$ for $0 < t < 6T$.

6.29. Refer to Figure 6.31, and let $R_g = 2\,Z_0$ and $R_L = 0.5Z_0$. Draw the current reflection diagram for $0 < t < 6T$, and plot I at $z = 3\ell/4$ for $0 < t < 6T$.

6.30. Refer to Figure 6.38a, and obtain an expression for I_-. Sketch $I_-(t)$ and $I(z)$ versus z for the time period $T < t < 2T$. The sketch should be similar to Figure 6.38b and Figure 6.38c.

6.31. Derive (6.58).

CHAPTER 6

6.1 $P_t = \frac{V^2}{2Z_0} \Rightarrow V = \sqrt{2P_t Z_0} = \sqrt{2\times10^4 \times 200} = 2000 \text{ V}$

6.2 Safety factor 10 $\Rightarrow |E|_{max} = 2\times10^7/10 = 2\times10^6 \text{ V/m} = V_0/a \Rightarrow V_0 = |E|_{max}\, a = 2\times10^6 \times \frac{4.11\times10^{-3}}{2} = 4$

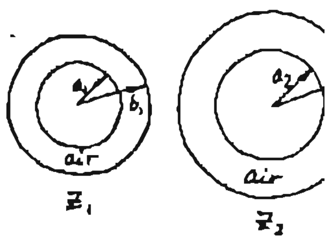
$P_t = \frac{\pi V_0^2}{\eta}\ln(b/a) = \frac{\pi\times(4110)^2\times\sqrt{1.5}}{120\pi}\times\ln(1.143/0.411) = 176.34 \text{ kW}$

6.3 Assume the breakdown E field of air is E_a, then $E_a = \frac{V_1}{a_1} = \frac{V_2}{a_2}$

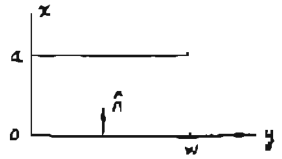
$Z_1 = Z_2 = 50\,\Omega \Rightarrow \frac{\eta_0}{2\pi}\ln(b_1/a_1) = \frac{\eta_0}{2\pi}\ln(b_2/a_2) \Rightarrow b_1/a_1 = b_2/a_2$

$P_1 = \frac{V_1^2}{2Z_1},\ P_2 = \frac{V_2^2}{2Z_2} \Rightarrow \frac{P_1}{P_2} = \frac{10^6}{10^3} = 1000 = \left(\frac{V_1}{V_2}\right)^2\left(\frac{Z_2}{Z_1}\right) = \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{a_1E_a}{a_2E_a}\right)^2 = \left(\frac{a_1}{a_2}\right)^2$

$\therefore \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{b_1}{b_2}\right)^2 = 1000 \Rightarrow a_1/a_2 = b_1/b_2 = \sqrt{1000} = 31.62$



6.4



$\bar{H} = \hat{y}(E_0/\eta)\,e^{-jkz}$

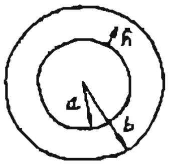
On the lower plate, $\bar{J}_s = \hat{n}\times\bar{H} = \hat{x}\times\hat{y}(E_0/\eta)e^{-jkz} = \hat{z}(E_0/\eta)e^{-jkz}$

$\therefore \underline{I} = \underline{J}_s W = W(E_0/\eta)e^{-jkz}$, same as (6.36)

6.5 $\bar{H} = \hat{\phi}\frac{V_0}{\eta\rho}e^{-jkz}$

On the inner conductor, $\bar{J}_s = \hat{n}\times\bar{H}\big|_{\rho=a} = \hat{\rho}\times\hat{\phi}\frac{V_0}{\eta a}e^{-jkz} = \hat{z}\frac{V_0}{\eta a}e^{-jkz}$

$\therefore \underline{I} = \underline{J}_s(2\pi a) = \frac{2\pi V_0}{\eta}e^{-jkz}$, same as (6.66) with $\alpha_z = 1$

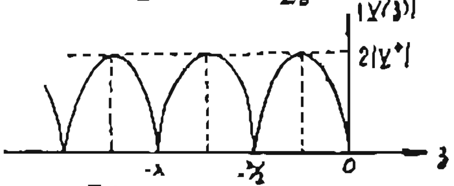


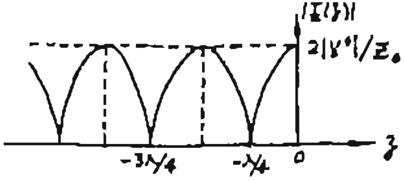
6.6 $Z_L = 0 \Rightarrow \Gamma_L = -1$

(a) $\underline{V}(z) = \underline{V}^+(e^{-jkz} + \Gamma_L e^{jkz}) = \underline{V}^+(e^{-jkz} - e^{jkz}) = -2j\underline{V}^+\sin(kz) \Rightarrow |\underline{V}(z)| = 2|\underline{V}^+||\sin(kz)|$

$\underline{I}(z) = \frac{1}{Z_0}\underline{V}^+(e^{-jkz} - \Gamma_L e^{jkz}) = \frac{1}{Z_0}\underline{V}^+(e^{-jkz} + e^{jkz}) = \frac{2}{Z_0}\underline{V}^+\cos(kz) \Rightarrow |\underline{I}(z)| = \frac{2}{Z_0}|V^+||\cos(kz)|$

(b)





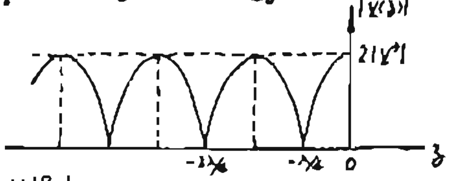
(c) $\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \infty$

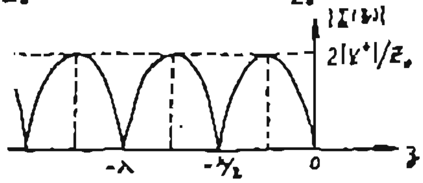
6.7 $Z_L = \infty \Rightarrow \Gamma_L = 1$

(a) $\underline{V}(z) = \underline{V}^+(e^{-jkz} + \Gamma_L e^{jkz}) = \underline{V}^+(e^{-jkz} + e^{jkz}) = 2\underline{V}^+\cos(kz) \Rightarrow |\underline{V}(z)| = 2|\underline{V}^+||\cos(kz)|$

$\underline{I}(z) = \frac{\underline{V}^+}{Z_0}(e^{-jkz} - \Gamma_L e^{jkz}) = \frac{\underline{V}^+}{Z_0}(e^{-jkz} - e^{jkz}) = -2j\frac{\underline{V}^+}{Z_0}\sin(kz) \Rightarrow |\underline{I}(z)| = \frac{2}{Z_0}|\underline{V}^+||\sin(kz)|$

(b)





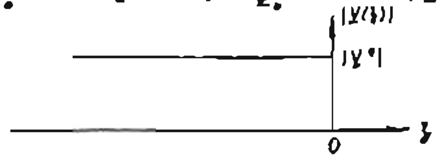
(c) $\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \infty$

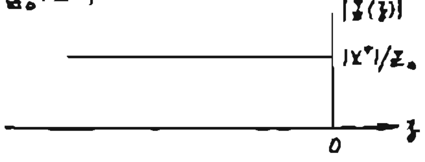
6.8 $Z_L = Z_0 \Rightarrow \Gamma_L = 0$

(a) $\underline{V}(z) = \underline{V}^+(e^{-jkz} + \Gamma_L e^{jkz}) = \underline{V}^+e^{-jkz} \Rightarrow |\underline{V}(z)| = |\underline{V}^+|$

$\underline{I}(z) = \frac{\underline{V}^+}{Z_0}(e^{-jkz} - \Gamma_L e^{jkz}) = \frac{\underline{V}^+}{Z_0}e^{-jkz} \Rightarrow |\underline{I}(z)| = \frac{1}{Z_0}|\underline{V}^+|$

(b)





(c) $\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 1$

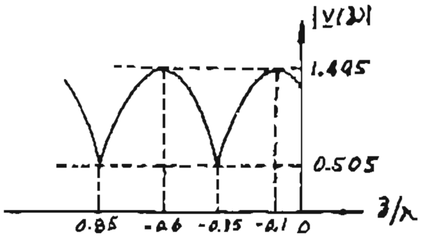
6.9 (a) $\Gamma_L = [(0.8+j1.0)-1]/[(0.8+j1.0)+1] = (j-0.2)/(j+1.8) = 0.495\angle 72.3^\circ \Rightarrow |\Gamma_L| = 0.495$

$VSWR = (1+|\Gamma_L|)/(1-|\Gamma_L|) = (1+0.495)/(1-0.495) = 2.96$

(b) $2kd_{min} = \phi_o + 180^\circ = 72.3^\circ + 180^\circ = 252.3^\circ \Rightarrow d_{min} = \frac{252.3}{2\times 360}\lambda = 0.35\lambda$

$\therefore$ minimum V at $z = -0.35\lambda$

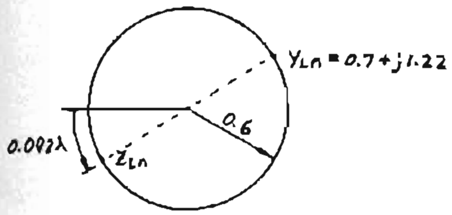
(c) $P_{ref}/P_{inc} = |\Gamma_L|^2 = (0.495)^2 = 0.245 \Rightarrow$ 24.5% of incident power is reflected

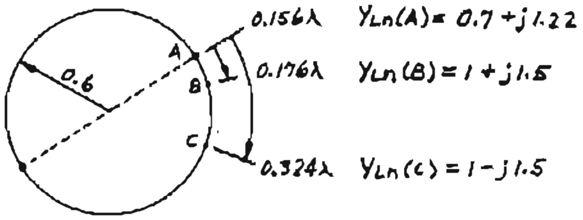


$|V(z)|^{max} = 1+|\Gamma_L| = 1+0.495 = 1.495$

$|V(z)|^{min} = 1-|\Gamma_L| = 1-0.495 = 0.505$

6.10





$Y_{Ln}(A) = 0.7+j1.22$

$Y_{Ln}(B) = 1+j1.5$

$Y_{Ln}(C) = 1-j1.5$

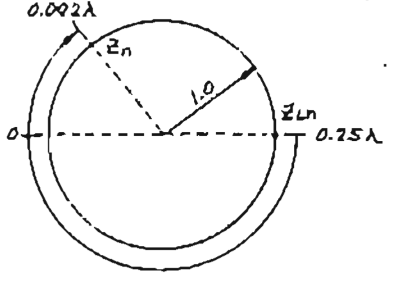
$Z_{Ln} = 0.34 - j0.59\ \Omega$

$\therefore Z_L = Z_{Ln}\times 50 = 17 - j29.5\ \Omega$

(ii) At $(0.176-0.156)\lambda = 0.02\lambda$ from load shunt $-j1.5$ ($= -j0.03$ mho)

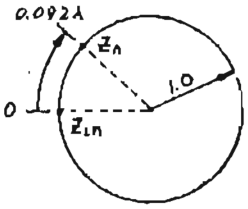
(iii) At $(0.324-0.156)\lambda = 0.168\lambda$ from load shunt $+j1.5$ ($= j0.03$ mho)

6.11 $Z_L = \infty \Rightarrow Z_{Ln} = \infty$; $Z(-\ell) = j33\Omega \Rightarrow Z_n(-\ell) = j0.66$



$\ell = 0.25\lambda + 0.092\lambda = 0.342\lambda$

6.12 $Z_L = 0 \Rightarrow Z_{Ln} = 0$; $Z_n(-\ell) = j0.66$



$\ell = 0.092\lambda$

6.13 $f = 8\,GHz \Rightarrow \lambda = \frac{3\times 10^8}{8\times 10^9} = 3.75$ cm

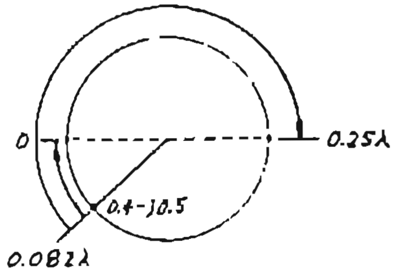
$b_o = 1.57 = \frac{4b}{\lambda}\ln\left[\csc\left(\frac{\pi d}{2b}\right)\right] = \frac{4\times 1.262}{3.75}\ln\left[\csc\left(\frac{\pi d}{2\times 1.262}\right)\right] \Rightarrow \ln\left[\csc\left(\frac{\pi d}{2.524}\right)\right] = \frac{1.57\times 3.75}{4\times 1.262} = 1.166$

$\therefore \csc\left(\frac{\pi d}{2.524}\right) = e^{1.166} = 3.21 \Rightarrow \frac{\pi d}{2.524} = \sin^{-1}\left(\frac{1}{3.21}\right) = 0.317 \Rightarrow d = \frac{0.317}{\pi}\times 2.542 = 0.25$ cm

6.14 From Smith Chart : (a) $0.707\angle 45^\circ$ (b) $1\angle 0^\circ$ (c) $1\angle 180^\circ$ (d) $0.37\angle -126^\circ$

6.15 From Smith Chart : (a) $1.25+j1.68$ (b) 0.53 (c) 1

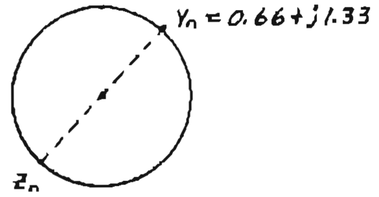
6.16



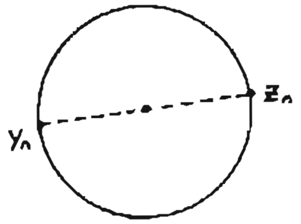
1st minimum at $z = -0.082\lambda$

1st maximum at $z = -(0.082 + 0.25)\lambda = -0.332\lambda$

6.17 (a)



(b)



$Y_n = 0.15 - j0.087$

6.18 $\Gamma_L = (Z_{Ln}-1)/(Z_{Ln}+1) = (1+j2)/(3+j2) = 0.62\angle 29.7^\circ = 0.54 + j0.31$

$\underline{V}^+ = 1 \Rightarrow |\underline{V}(z)| = |1 + \Gamma_L e^{j2kz}|$; $Z_o = 1 \Rightarrow |\underline{I}(z)| = |1 - \Gamma_L e^{j2kz}|$

$\underline{V}_{max} = |1 + 0.62| = 1.62$ and $\underline{I}_{max} = 1.62$; $\underline{V}_{min} = |1 - 0.62| = 0.38$ and $\underline{I}_{min} = 0.38$

$|\underline{V}(0)| = |1 + \Gamma_L| = |1 + (0.54 + j0.31)| = |1.54 + j0.31| = 1.57$

$|\underline{V}(-0.219\lambda)| = |1 + (0.62\angle 29.7^\circ)e^{j\frac{4\pi}{\lambda}(-0.219\lambda)}| = |1 + 0.62\angle 29.7^\circ - 157.7^\circ| = |1 + 0.62\angle -128^\circ| = 0.79$

$|\underline{I}(0)| = |1 - \Gamma_L| = |1 - (0.54 + j0.31)| = |0.46 - j0.31| = 0.55$

$|\underline{I}(-0.219\lambda)| = |1 - 0.62\angle -128^\circ| = 1.47$

6.19 From Problem 6.18, $|\underline{V}(-0.219\lambda)| = 0.79$

After shunt admittance is added, $|\underline{V}(-0.219\lambda)| = 1$

$\Rightarrow$ $\underline{V}$ and $\underline{I}$ should be multiplied by a factor $1/0.79 = 1.266$

$\therefore$ After shunt, $\underline{V}_{max} = 1.62 \times 1.266 = 2.05$; $|\underline{V}(0)| = 1.57 \times 1.266 = 2.00$;

$\underline{I}_{max} = |\underline{I}(-0.219\lambda)| \times 1.266 = 1.47 \times 1.266 = 1.86$;

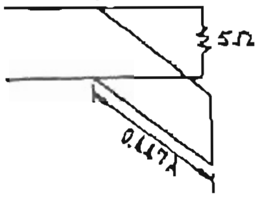
$|\underline{I}(0)| = 0.55 \times 1.266 = 0.70$; and $\underline{I}_{min} = 0.38 \times 1.266 = 0.48$

6.20 $\lambda = 19.8$ cm; at the point B in Figure 6.24, $l_1 = (0.25 + 0.201)\lambda = 0.451\lambda = 8.93$ cm

input admittance of the stub should be $Y_B = -j2.9 \Rightarrow Z_B = j0.345$

From Smith Chart, $l_2 = 0.0524\lambda = 1.04$ cm

6.21

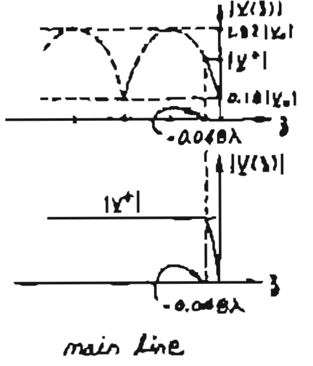


on the main line, $\Gamma_L = (5-50)/(5+50) = -0.82$ and $|\underline{V}(z)| = |\underline{V}_0||1 - 0.82e^{j2k}$

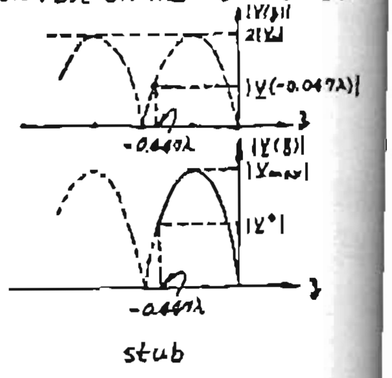
on the stub, $\Gamma_L = -1$ and $|\underline{V}(z)| = 2|\underline{V}_0||\sin kz|$

after $z = -0.048\lambda$ on the main line. line is matching and $\Gamma_L = 0$,

$\therefore |\underline{V}(z)| = |\underline{V}(-0.048\lambda)| = |\underline{V}^+|$ for $z \le -0.048\lambda$ on the main line.



main line



stub

breakdown $E = 1.81\times10^{5}$ V/cm $= 1.81\times10^{7}$ V/m ; use safety factor 10, then $E^{max} = 1.81\times10^{6}$ V/m

For coaxial line, $E^{max} = |\underline{V}_{max}|/\{a\ln(b/a)\} \rightarrow |\underline{V}_{max}| = 1.81\times10^{6}\times2\times10^{-3}\times1.264 = 4576$ V

On the stub. $|\underline{V}(-0.047\lambda)| = 2|\underline{V}_0||\sin\frac{2\pi}{\lambda}(-0.447\lambda)| = 0.654|\underline{V}_0|$

$\therefore |\underline{V}^+|/|\underline{V}_{max}| = |\underline{V}(-0.447\lambda)|/2|\underline{V}_0| \Rightarrow |\underline{V}^+| = |\underline{V}_{max}||\underline{V}(-0.447\lambda)|/2|\underline{V}_0| = 4576\times0.654/2 = 1496$ V

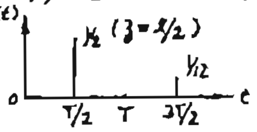
$\therefore P^{max} = |\underline{V}^+|^2/2Z_0 = (1496)^2/(2\times50) = 22.4$ kW

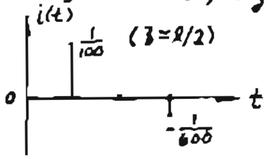
6.22 $V^+ = 50/(50+50) = 1/2$, $I^+ = V^+/50 = 0.01$, $\Gamma_L = (70-50)/(70+50) = 0.167$, $\Gamma_g = 0$

$T = l/v = 2\times10^{-8}$ sec

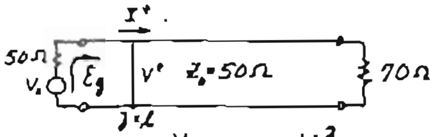
$V^- = \Gamma_L V^+ = 0.167\times(\frac{1}{2}) = 0.0835$

$I^- = -\Gamma_L I^+ = -0.167\times(0.01) = -0.00167$





6.23



$V^+ = I^+Z_0 = \frac{V_0}{50+50}\times50 = V_0/2$

$\Gamma_L = (70-50)/(70+50) = 1/6$

$\mathcal{E}_g = V_0 I^+\Delta = V_0\frac{V_0}{100}\Delta = \frac{V_0^2}{100}\Delta$, at $z=0$

$\mathcal{E}^+ = V^+I^+\Delta = \frac{V_0}{2}\cdot\frac{V_0}{100}\Delta = \frac{V_0^2}{200}\Delta$, at $z=l$

$\mathcal{E}_{ref} = (V^+\Gamma_L)(I^+\Gamma_L)\Delta = \frac{V_0}{12}\cdot\frac{V_0}{600}\Delta = \frac{V_0^2}{7200}\Delta$, at $z=l$

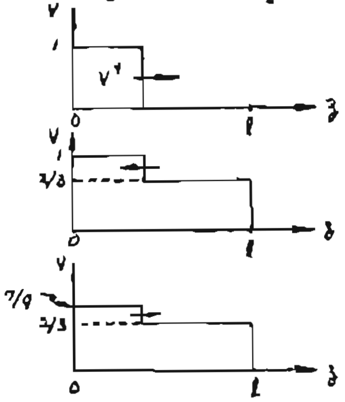
$\mathcal{E}_{loss} = \mathcal{E}^+ - \mathcal{E}_{ref} = (\frac{1}{200}-\frac{1}{7200})V_0^2\Delta = \frac{35}{7200}V_0^2\Delta \Rightarrow \mathcal{E}_l/\mathcal{E}_g = (\frac{35}{7200}V_0^2\Delta)/(\frac{1}{100}V_0^2\Delta) = \frac{35}{72} = 48.6\%$

6.24 $1:\Gamma:\Gamma^2:\Gamma^3 = 2^3:2^2:2^1:2^0 \Rightarrow \Gamma = \frac{1}{2}$ $\quad\therefore R = Z_0\frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3Z_0$

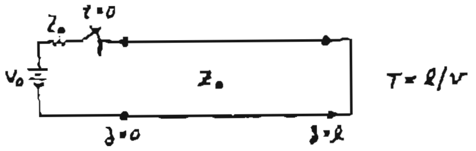
Sampler should be located at the middle between load and generator.

Sampling time at $t = (n+\frac{1}{2})T$ with $n \geq 3$.

6.25 $\Gamma_L = \Gamma_g = (0.5-1)/(0.5+1) = -\frac{1}{3} \Rightarrow V^- = \Gamma_L V^+ = -\frac{1}{3}V^+$, $(V^+)' = \Gamma_g V^- = \Gamma_L\Gamma_g V^+ = \frac{1}{9}V^+$



6.26



$T = l/v$

$\Gamma_g = 0$

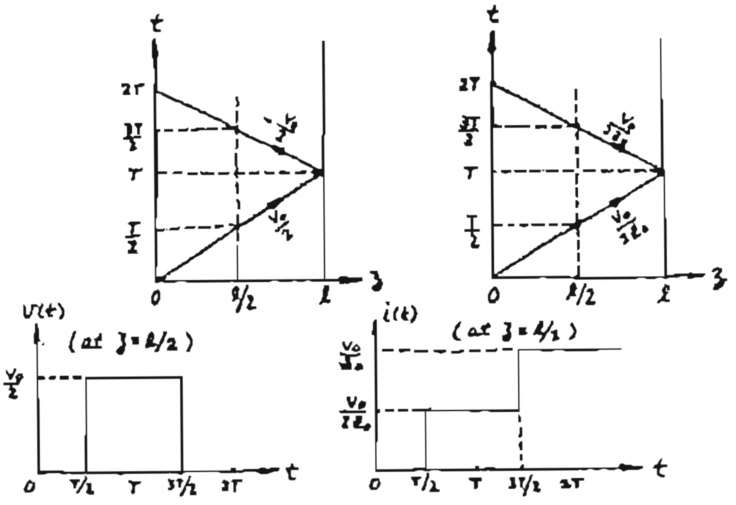
$\Gamma_L = -1$

$V_+ = V_0\frac{Z_0}{Z_0+Z_0} = \frac{V_0}{2}$

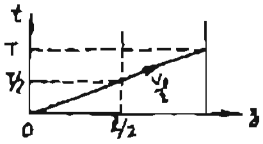
$V_- = \Gamma_L V_+ = -\frac{V_0}{2}$

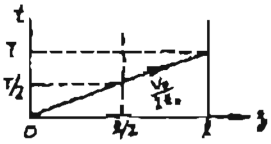
$I_+ = V_+/Z_0 = \frac{V_0}{2Z_0}$

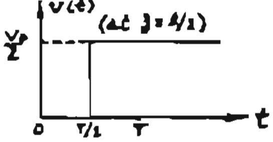
$I_- = -\Gamma_L I_+ = \frac{V_0}{2Z_0}$

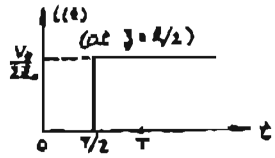


6.27 $V_+ = V_0 Z_0/(Z_0+Z_0) = \frac{V_0}{2}$, $I_+ = V_+/Z_0 = \frac{V_0}{2Z_0}$; $\Gamma_g = \Gamma_L = 0 \Rightarrow V_- = 0$ and $I_- = 0$



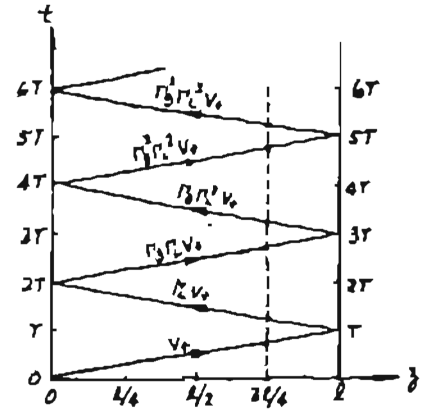


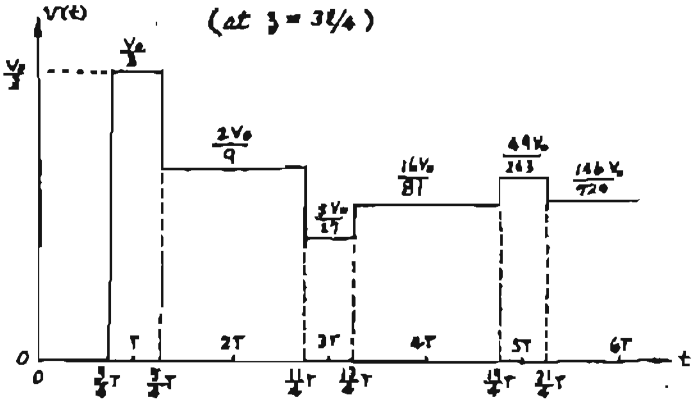




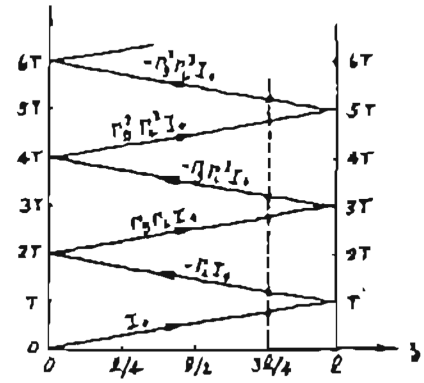
6.28 $\Gamma_g = (2Z_0 - Z_0)/(2Z_0+Z_0) = \frac{1}{3}$, $\Gamma_L = (0.5Z_0 - Z_0)/(0.5Z_0 + Z_0) = -\frac{1}{3}$

$V_+ = \frac{V_0}{R_g + Z_0} \cdot Z_0 = \frac{V_0}{3}$, $I_+ = \frac{V_0}{Z_0} = \frac{V_0}{3Z_0}$





6.29


6T
5T
4T
3T
2T
T
0
$-\Gamma_g^3\Gamma_L^3 I_+$
$\Gamma_g^2\Gamma_L^2 I_+$
$-\Gamma_g\Gamma_L^2 I_+$
$\Gamma_g\Gamma_L I_+$
$-\Gamma_L I_+$
I_+
6T
5T
4T
3T
2T
T
0
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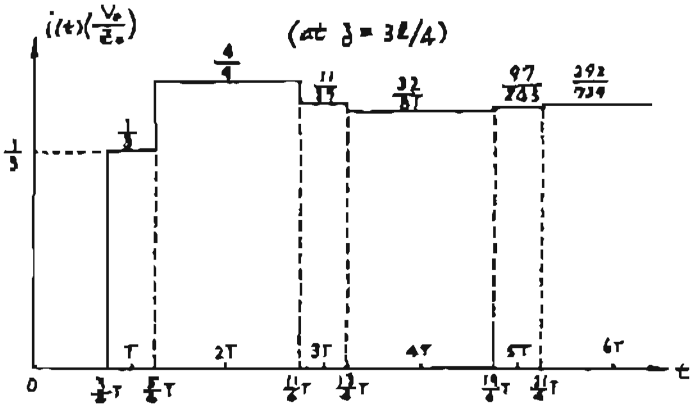
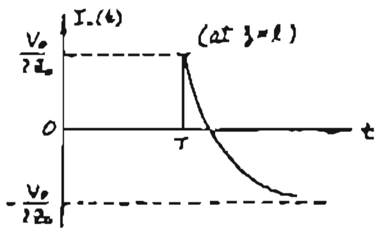
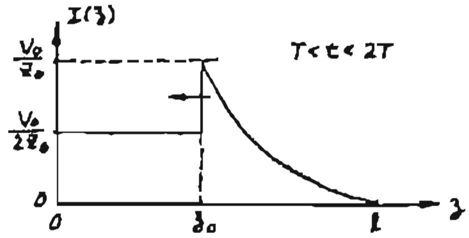
ℓ
z



6.30 $I_- = -\frac{V_-}{Z_0} = -\frac{V_0}{2Z_0}\left[1 - 2e^{-(t-T)/Z_0C}\right]$, $T = \ell/v$





6.31 $V_+ = V_0/2$ and $I_+ = V_+/Z_0 = \frac{V_0}{2Z_0}$

at $z = \ell$, $V_L = V_+ + V_-$ and $I_L = I_+ + I_- = (V_+ - V_-)/Z_0 \Rightarrow V_+ - V_- = I_L Z_0$

$\therefore\ 2V_+ = V_L + Z_0 I_L \Rightarrow V_L = 2V_+ - Z_0 I_L$

Also, $\frac{dI_L}{dt} = \frac{V_L}{L} = \frac{1}{L}(2V_+ - Z_0 I_L) \Rightarrow \frac{dI_L}{dt} + \frac{Z_0}{L} I_L = \frac{2V_+}{L} \Rightarrow I_L(t) = \frac{2V_+}{Z_0} + Ae^{-\frac{Z_0}{L}t}$

using $I_L(t=T) = 0$, $A = -\frac{2V_+}{Z_0} e^{\frac{Z_0}{T}t} \Rightarrow I_L(T<t<2T) = \frac{2V_+}{Z_0}\left[1 - e^{-\frac{Z_0}{L}(t-T)}\right]$

$I_- = I_L - I_+ = \frac{2V_+}{Z_0} - \frac{2V_+}{Z_0} e^{-\frac{Z_0}{L}(t-T)} - \frac{V_+}{Z_0} = \frac{V_+}{Z_0}\left[1 - 2e^{-\frac{Z_0}{L}(t-T)}\right]$

$\therefore\ V_- = -I_- Z_0 = -V_+\left[1 - 2e^{-\frac{Z_0}{L}(t-T)}\right] = V_0\left[e^{-\frac{Z_0}{L}(t-T)} - \frac{1}{2}\right]$

to Puerto Rico. This radio telescope system is called the **Very Long Baseline Array**. The angular resolution will be of the order of 10^{-9} radian, which is smaller than the angle spanned by a dime located in New York City when it is viewed from Los Angeles.

In ordinary arrays, individual antenna elements are connected by transmission lines. For the very long baseline array, such physical interconnection of antennas is not practical. Instead, the signal received by each antenna in the array is recorded on magnetic tapes which are later transported to a central facility where the tapes are replayed simultaneously. The key to such processing is a very accurate time standard for all recorded data. At present, time synchronization of the recordings is provided by hydrogen masers which are accurate within 20 nanoseconds*.

Problems

7.1 Find the rectangular coordinates of a point P where the spherical coordinates are $(r = 1, \theta = 60°, \phi = 30°)$.

7.2 The rectangular coordinates of a point Q are $(1, 2, -4)$. Find its spherical coordinates.

7.3 Show that $\nabla \cdot \nabla \times \mathbf{A} = 0$ in spherical coordinates for any vector $\mathbf{A}$.

7.4 Show that the differential spherical surface element is equal to $ds = r^2 \sin\theta \, d\theta \, d\phi$. **Hint:** Refer to Figure P7.4.

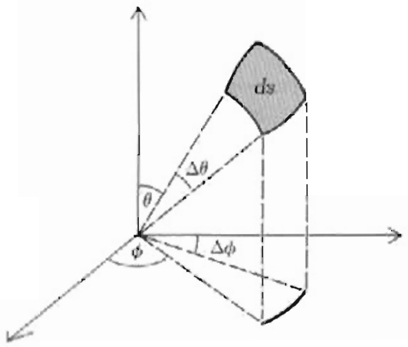


Figure P 7.4

7.5 To convert a vector in spherical coordinates to the same in rectangular coordinates, it is convenient to prepare a table for dot products between unit vectors in these coordinate systems. For example, $\hat{x} \cdot \hat{r} = \sin\theta\cos\phi$, as indicated in the following table. Complete this table.

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*K. I. Kellermann and A. R. Thompson, "The very long baseline array," Science, Vol. 229, No. 4709, July 1985, pp. 123–130.

$\cdot$	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}$	$\sin\theta\cos\phi$		
$\hat{y}$			
$\hat{z}$			

7.6 Use the table prepared in the preceding problem to express the following vector A located at $(r = 1, \theta = -60°, \phi = 45°)$ in rectangular coordinates:

$$A = 12\hat{r} + 8\hat{\theta} - 5\hat{\phi}$$

7.7 Show that the distance function $|\mathbf{r} - \mathbf{r}'|$ that appears in (7.7) and (7.8) can be expressed in spherical coordinates as

$$|\mathbf{r} - \mathbf{r}'|^2 = r^2 + r'^2 - 2rr'\cos\gamma$$

$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi')$$

where γ is the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$ and (r, θ, ϕ) and (r', θ', ϕ') are spherical coordinates of $\mathbf{r}$ and $\mathbf{r}'$, respectively.

7.8 A vertical receiving dipole antenna at P is 15 km away from a capacitor-plate antenna that is also placed vertically, as shown in Figure P7.8. The receiving antenna measures an E field equal to 10 mV/m. What is the value of E that the same receiving antenna will detect at a height 3 km above P? What must the orientation of the receiving dipole be to obtain a maximum reading? (A maximum reading is obtained if the dipole is parallel to the E field.)

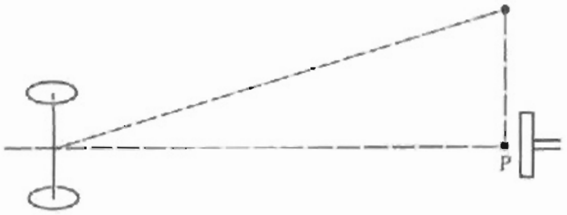


Figure P 7.8

7.9 The power lost on a cylindrical conductor that is Δz long and that carries I amperes of current is given by

$$P_{\text{ohm}} = \tfrac{1}{2} I^2 R_s \Delta z$$

where P_{ohm} is the loss due to finite conductivity of the wire, R_s is the surface resistance given by $1/(\sigma d_s 2\pi a)$, and d_s is the skin depth. The efficiency of the antenna is given by

$$\eta_e = \frac{\text{Power radiated}}{\text{Power radiated} + P_{\text{ohm}}}$$

Assume that a short antenna of length Δz has an efficiency of ten percent. Is the efficiency improved by increasing the length to $2\,\Delta z$ while maintaining the same current and, if so, by how much? Assume that the antenna is still a short antenna after its length is increased to $2\,\Delta z$.

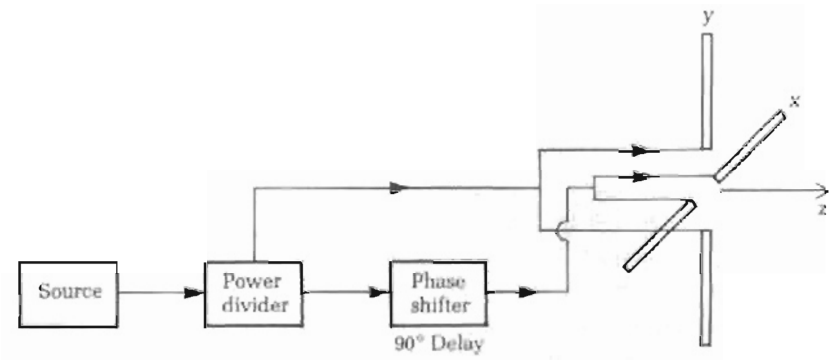


Figure P 7.10

7.10. Consider the antenna system consisting of two short dipoles arranged perpendicularly to each other in space, as shown in Figure P7.10. These dipoles are driven by the same amount of power from a common source. However, the current on the $\hat{x}$-oriented dipole has a $-90°$ phase with respect to that on the $\hat{y}$-oriented dipole because of a phase shifter inserted in the transmission line that leads to the former. Find the total radiated electric field on the z axis. Verify that this antenna system radiates a circularly polarized wave in the $\hat{z}$ direction. Is the wave left-hand or right-hand circularly polarized?

7.11. Find the expression of the total radiated electric field on the x axis that is due to the antenna system discussed in the preceding problem. What is its polarization on the x axis?

7.12. A certain application requires that a field strength of 1 V/m be maintained at a point 1 km from an antenna located in free space. What power must be fed to the antenna if it is (a) an isotropic antenna, (b) a short dipole, and (c) a half-wave dipole? Neglect ohmic loss. An isotropic antenna radiates an equal amount of power in all directions.

7.13. The current at the center of an antenna is 100 A; what is the E field 1 km away from it on the horizontal ($\theta = 90°$) plane at 10 MHz if the antenna is (a) a dipole with $h_1 = h_2 = 0.5$ m, (b) a capacitor-plate antenna with $\Delta z = 1$ m, and (c) a half-wave dipole?

7.14. Show that if the radiation field pattern shown in Figure 7.4 for the infinitesimal dipole or the capacitor-plate antenna is plotted in x-z plane in linear scale the pattern is exactly formed by two circles.

7.15. Find the directivity of (a) an isotropic antenna, (b) a capacitor-plate antenna, and (c) a half-wave dipole.

7.16. Find the radiated electric field of a linear antenna that is 3 m long ($\ell = 3$ m) and that operates at 100 MHz in air. Plot its radiation pattern.

7.17. Consider a uniform linear array of two half-wave dipoles that are 1.5 wavelengths apart. The currents on these two dipoles are in phase. Sketch the radiation pattern in the horizontal ($\theta = 90°$) plane. Show clearly the number of lobes in this pattern. Also, estimate the beam width of each of the major lobes. The beam width is the angle between two directions in which the radiation intensity is one-half (-3 dB) the maximum value of the beam.

7.18. Figure 7.23(b) shows the array factor of a two-element array separated by 20λ. Find the beam width (in terms of the angle between two adjacent nulls) of this array factor near $\phi = 90°$ and $\theta = 30°$.

(a) Use the approximate formula given by (7.49).
(b) Find the exact value starting from (7.45).

7.19. Find the directivity of the two-wire transmission line shown in Figure 7.27 with radiation fields given by (7.42).

7.20. Find the field pattern of a two-element array with $d = \lambda/4$ and $\psi = 0$. Sketch the field pattern on the x-y plane.

7.21. Find the field pattern of a four-element array with $d = \lambda/4$ and $\psi = 0$. Sketch the field pattern on the x-y plane. (a) Use (7.37) to obtain the field-pattern formula, and (b) use the result obtained in the preceding problem and in Figure 7.16 and the pattern-multiplication technique.

7.22. Write a computer program to plot field patterns of a ten-element phased array with $d = \lambda/4$ and varying phases.

7.23. A uniform linear array consists of 6 short dipoles. The spacing between adjacent elements is λ/4, as shown in Figure P7.23.

(a) What should the phase shift ψ be, in order to point the maximum radiation in the $\phi = 90°$ (that is, $\hat{y}$) direction?
(b) Suppose that the E-field due to the first element (the dipole at far left) is given as follows:

$$E_{\theta a} = \frac{1000}{r} e^{-jkr} \sin\theta$$

Calculate $|E_\theta|$ of the entire array at point A(0,1000,0), point B(1000,0,0), point C(0, − 1000,0), and point D(− 1000,0,0), separately. All positions are given in rectangular coordinates in meters. Use the phase shift found in (a).

(c) Sketch the field pattern of the array in the x-y plane.
(d) Sketch the field pattern of the array in the x-z plane.

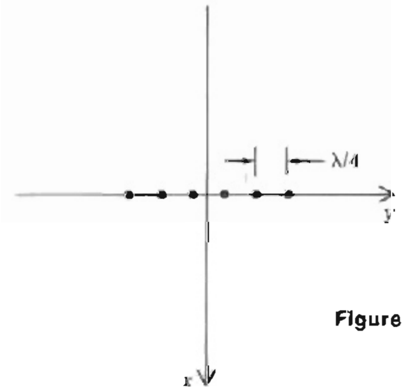


Figure P7.23

CHAPTER 7

7.1 $x = 1\cdot\sin 60^\circ\cos 30^\circ = 3/4$, $y = 1\cdot\sin 60^\circ\sin 30^\circ = \sqrt{3}/4$, $z = 1\cdot\cos 60^\circ = 1/2$

7.2 $r = \sqrt{1+(2)^2+(-4)^2} = \sqrt{21}$, $\phi = \tan^{-1}(2/1) = 63.4^\circ$, $\theta = \cos^{-1}\left(\frac{-4}{\sqrt{21}}\right) = 150.8^\circ$

7.3
$$\nabla\cdot\nabla\times\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left\{\frac{r}{\sin\theta}\left[\cos\theta A_\phi + \sin\theta\frac{\partial A_\phi}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left\{\frac{1}{r}\frac{\partial A_r}{\partial\phi} - \frac{\sin\theta}{r}A_\phi - \sin\theta\frac{\partial A_\phi}{\partial r}\right\} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\phi}\left\{r\frac{\partial A_\theta}{\partial r} + A_\theta - \frac{\partial A_r}{\partial\theta}\right\}$$
$$= \frac{1}{r^2}\left\{\cot\theta\, r\frac{\partial A_\phi}{\partial r} + \cot\theta A_\phi + r\frac{\partial^2 A_\phi}{\partial r\partial\theta} + \frac{\partial A_\phi}{\partial\theta} - \frac{r}{\sin\theta}\frac{\partial^2 A_\theta}{\partial r\partial\phi} - \frac{1}{\sin\theta}\frac{\partial A_\theta}{\partial\phi}\right\} + \frac{1}{r^2\sin\theta}\frac{\partial^2 A_r}{\partial\theta\partial\phi} - \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial A_\phi}{\partial\theta} + \cos\theta A_\phi\right]$$
$$-\frac{1}{r}\cot\theta\frac{\partial A_\phi}{\partial r} - \frac{1}{r}\frac{\partial^2 A_\phi}{\partial r\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial^2 A_\theta}{\partial\phi\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial A_\theta}{\partial\phi} - \frac{1}{r^2\sin\theta}\frac{\partial^2 A_r}{\partial\phi\partial\theta}$$
$$= \cancel{\frac{1}{r}\cot\theta\frac{\partial A_\phi}{\partial r}} + \cancel{\frac{1}{r^2}\cot\theta A_\phi} + \cancel{\frac{1}{r}\frac{\partial^2 A_\phi}{\partial r\partial\theta}} + \cancel{\frac{1}{r^2}\frac{\partial A_\phi}{\partial\theta}} - \cancel{\frac{1}{r\sin\theta}\frac{\partial^2 A_\theta}{\partial r\partial\phi}} - \cancel{\frac{1}{r^2\sin\theta}\frac{\partial A_\theta}{\partial\phi}} + \cancel{\frac{1}{r^2\sin\theta}\frac{\partial^2 A_r}{\partial\theta\partial\phi}} - \cancel{\frac{1}{r^2}\frac{\partial A_\phi}{\partial\theta}} - \cancel{\frac{1}{r^2}\cot\theta A_\phi}$$
$$-\cancel{\frac{1}{r}\cot\theta\frac{\partial A_\phi}{\partial r}} - \cancel{\frac{1}{r}\frac{\partial^2 A_\phi}{\partial r\partial\theta}} + \cancel{\frac{1}{r\sin\theta}\frac{\partial^2 A_\theta}{\partial\phi\partial r}} + \cancel{\frac{1}{r^2\sin\theta}\frac{\partial A_\theta}{\partial\phi}} - \cancel{\frac{1}{r^2\sin\theta}\frac{\partial^2 A_r}{\partial\phi\partial\theta}} \equiv 0$$

7.4 $ds = (r\Delta\theta)(r\sin\theta\Delta\phi) = r^2\sin\theta\,\Delta\theta\Delta\phi$

7.5

$\cdot$	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{y}$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{z}$	$\cos\theta$	$-\sin\theta$	0

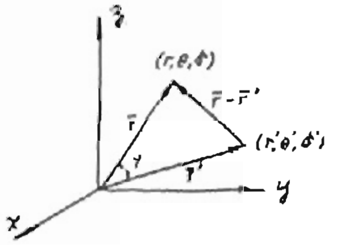
7.6 $A_x = \hat{x}\cdot\bar{A} = 12\sin\theta\cos\phi + 8\cos\theta\cos\phi + 5\sin\phi = 12\sin(-60^\circ)\cos(45^\circ) + 8\cos(-60^\circ)\cos(45^\circ) + 5\sin(45^\circ) = -0.9845$

$A_y = \hat{y}\cdot\bar{A} = 12\sin\theta\sin\phi + 8\cos\theta\sin\phi - 5\cos\phi = 12\sin(-60^\circ)\sin(45^\circ) + 8\cos(-60^\circ)\sin(45^\circ) - 5\cos(45^\circ) = -8.056$

$A_z = \hat{z}\cdot\bar{A} = 12\cos\theta - 8\sin\theta = 12\cos(-60^\circ) - 8\sin(-60^\circ) = 12.93$

$\therefore \bar{A} = \hat{x}(-0.9845) + \hat{y}(-8.056) + \hat{z}\,12.93$

7.7



$$|\bar{r}-\bar{r}'|^2 = |\bar{r}|^2 + |\bar{r}'|^2 - 2|\bar{r}||\bar{r}'|\cos\gamma$$
$$= r^2 + r'^2 - 2rr'\cos\gamma$$
$$\cos\gamma = \hat{r}\cdot\hat{r}' = (\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta)\cdot(\hat{x}\cos\theta'\cos\phi' + \hat{y}\sin\theta'\sin\phi' + \hat{z}\cos\theta')$$
$$= \sin\theta\cos\phi\sin\theta'\cos\phi' + \sin\theta\sin\phi\sin\theta'\sin\phi' + \cos\theta\cos\theta'$$
$$= \sin\theta\sin\theta'(\cos\phi\cos\phi' + \sin\phi\sin\phi') + \cos\theta\cos\theta'$$
$$= \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi')$$

7.8 For the capacitor plate antenna, $\underline{E} = \eta\frac{jk(I\Delta z)e^{-jkr}}{4\pi r}\sin\theta\,\hat{\theta} \Rightarrow |\underline{E}| = A\left|\frac{\sin\theta}{r}\right|$

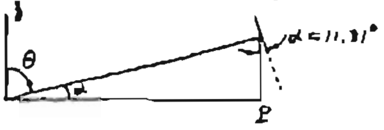
At P, $r = 15000$ and $\theta = 90^\circ$, $|\bar{E}| = 10$ mV/m $\Rightarrow 10\times10^{-3} = A\left(\frac{1}{15000}\right) \Rightarrow A = 150$

$\therefore |\underline{E}(r,\theta)| = 150\left|\frac{\sin\theta}{r}\right|$

At 3 km above P, $r = \sqrt{(15000)^2+(3000)^2} = 15297.1$ m and $\alpha = \tan^{-1}(3000/15000) = 11.31^\circ$

$\therefore |\underline{E}| = 150\left|\frac{\sin(90^\circ-11.31^\circ)}{15297.1}\right| = 9.62$ mV/m

Maximum reading is obtained when dipole makes 11.31° from vertical



7.9 $P_{rad} \sim (\Delta z)^2$; $P_{ohm} \sim (\Delta z)$

at length Δz, $\eta_a = 0.1 = P_{rad}/(P_{rad}+P_{ohm}) \Rightarrow P_{ohm} = 9P_{rad}$

at length $2\Delta z$, $\eta_a = 4P_{rad}/(4P_{rad}+18P_{rad}) = \frac{4}{22} = 18.2\%$

7.10 on the z-axis due to $\hat{y}$ directed dipole $\underline{E}_1 = \frac{jk\eta(I_1\Delta l)}{8\pi z}e^{-jkz}(-\hat{y})$ $\quad(I_2 = e^{-j90^\circ}I_1 = -jI_1)$

due to $\hat{x}$ directed dipole $\underline{E}_2 = \frac{jk\eta(I_2\Delta l)}{8\pi z}e^{-jkz}(-\hat{x}) = \frac{jk\eta(I_1\Delta l)}{8\pi z}e^{-jkz}(-j)(-\hat{x})$

$\underline{E} = \underline{E}_1 + \underline{E}_2 = \frac{jk\eta(I_1\Delta l)}{8\pi z}e^{-jkz}(j\hat{x}-\hat{y})$

Since $\frac{|\underline{E}_1|}{|\underline{E}_2|} = \frac{|\underline{E}_y|}{|\underline{E}_x|} = 1$ and $\phi = 90^\circ$, it is left-hand circularly polarized.

7.11 on the x-axis due to $\hat{x}$ oriented dipole. $\bar{E}_1 = 0$

due to $\hat{y}$ oriented dipole, $\bar{E}_2 = \frac{jk\eta(I\Delta l)}{8\pi x} e^{-jkx}(-\hat{y})$

$\bar{E} = \bar{E}_1 + \bar{E}_2 = (-\hat{y}) \frac{jk\eta(I\Delta l)}{8\pi x} e^{-jkx}$, linearly polarized

7.12 (a) $|\langle \bar{S} \rangle| = |\frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}| = \frac{1}{2\eta}|\bar{E}|^2 = \frac{1}{240\pi}$

$P = \frac{1}{240\pi} \times 4\pi \times (1000)^2 = 16.67$ KW

(b) For a short dipole, $|\bar{E}| = \eta \frac{kI\Delta z}{4\pi r} \Rightarrow \eta \frac{kI\Delta z}{4\pi} = |\bar{E}| r = 1 \times 1000 = 1000$

$P = \frac{4\pi}{3}\eta \left|\frac{kI\Delta z}{4\pi}\right|^2 = \frac{4\pi}{3\eta}\left(\eta\left|\frac{kI\Delta z}{4\pi}\right|\right)^2 = \frac{4\pi}{3\eta}(|\bar{E}|r)^2 = \frac{4\pi}{3 \times 120\pi} \times (1000)^2 = 11.1$ KW

(c) For a half-wave dipole, $|\langle \bar{S} \rangle| = \frac{1}{240\pi} = \frac{I_0^2 \eta}{8\pi^2 r^2} = \frac{120\pi \times I_0^2}{8\pi^2 (1000)^2} \Rightarrow I_0^2 = \frac{8\pi^2 \times 10^6}{120\pi \times 240\pi} = 277.78$

$P = \frac{I_0^2 \eta \, C_{in}(2\pi)}{8\pi} = \frac{277.78 \times 120\pi}{8\pi} \times 2.4376 = 10.16$ KW

7.13 $f = 10$ MHz $\Rightarrow \lambda = \frac{3 \times 10^8}{10 \times 10^6} = 30$ m

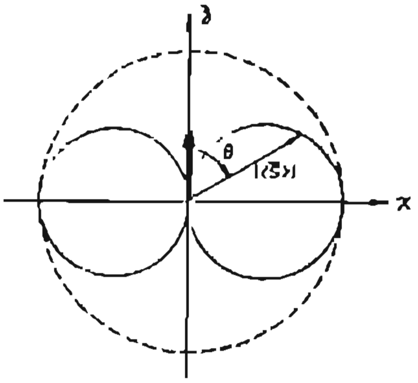
(a) short dipole, $I = 100/2 = 50$ A and $|\bar{E}| = \frac{\eta k I \Delta h}{4\pi r} = \frac{120\pi \times (2\pi/30) \times 50 \times (0.5 + 0.5)}{4\pi \times 1000} = 0.314$ V/m

(b) capacitor-plate antenna, $I = 100$A and $|\bar{E}| = 0.314 \times 2 = 0.628$ V/m

(c) half-wave dipole, $|\bar{E}| = \frac{I\eta}{2\pi r} = \frac{100 \times 120\pi}{2\pi \times 1000} = 6$ V/m

7.14 $\langle \bar{S} \rangle = \hat{r} \frac{\eta}{2} \left[\frac{kI\Delta z}{4\pi r}\right]^2 \sin^2\theta = \hat{r} A \sin^2\theta \Rightarrow$ normalized $\langle \bar{S} \rangle = \hat{r} \sin^2\theta$

θ	±0°	±10°	±20°	±30°	±40°	±50°	±60°	±70°	±80°	±90°
$\sin^2\theta$	0	0.03	0.17	0.25	0.41	0.59	0.75	0.88	0.97	1.00



7.15 (a) $D = 1$

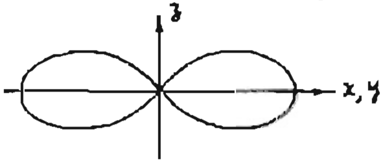
(b) $D = 1.5$

(c) $D(\theta) = \frac{|I_0|^2 \eta \cos^2[(\pi/2)(\cos\theta)]}{8\pi^2 r^2 \sin^2\theta} \times 4\pi r^2 \times \frac{8\pi}{|I_0|^2 \eta \, C_{in}(2\pi)} = \frac{4\cos^2[(\pi/2)\cos\theta]}{\sin^2\theta \, C_{in}(2\pi)}$

$\therefore D = D(90^\circ) = \frac{4}{2.4376} = 1.64$

7.16 $f = 100$ MHz $\Rightarrow \lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3$ m and $l = 3\text{m} = \lambda \Rightarrow kl = (2\pi/\lambda) \cdot l = 2\pi$

$\bar{E} = \hat{\theta}\eta \frac{jI_0 e^{-jkr}}{2\pi r \sin\theta}[\cos((kl/2)\cos\theta) - \cos(kl/2)] = \hat{\theta}\eta \frac{jI_0 e^{-jkr}}{2\pi r} \cdot \frac{1}{\sin\theta} \cdot [\cos(\pi\cos\theta) + 1]$



7.17 $d = 1.5\lambda$, $kd = \frac{2\pi}{\lambda} \times 1.5\lambda = 3\pi$; $\theta = 90^\circ$, $\cos\gamma = \sin\phi$; $N = 2$; $\psi = 0$

$|F(\phi)| = \left|\cos\left(\frac{kd\cos\gamma + \psi}{2}\right)\right| = \left|\cos\left(\frac{3\pi}{2}\sin\phi\right)\right|$

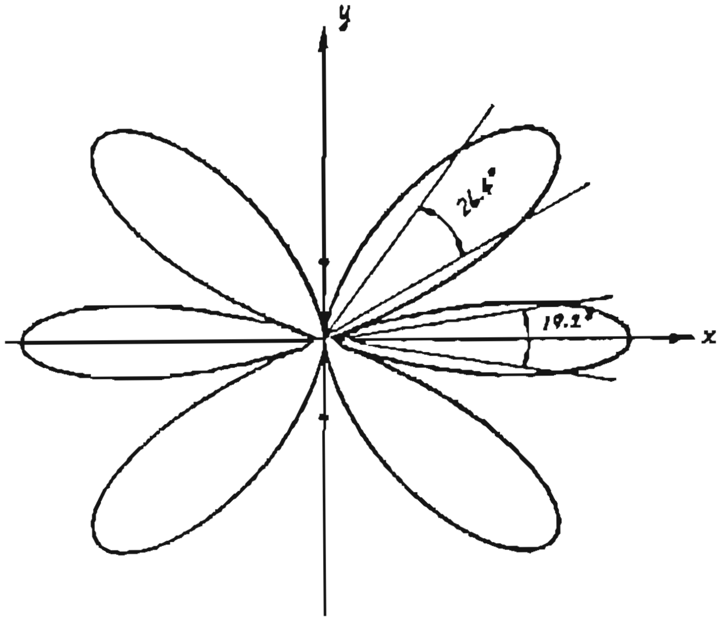
$|F(\phi)| = 1$ at $\phi = 0^\circ$ and $\phi = 41.8^\circ$ $\qquad -3\text{dB} = 20\log(E/E_{max}) \Rightarrow \frac{E}{E_{max}} = 10^{-0.15} = 0.708$

$|F(\phi)| = 0.708 = \left|\cos\left(\frac{3\pi}{2}\sin\phi\right)\right| \Rightarrow \frac{3\pi}{2}\sin\phi_1 = \frac{\pi}{4}$, $\phi_1 = \sin^{-1}\left(\frac{1}{6}\right) = 9.6^\circ$

$$\Rightarrow \frac{3\pi}{2}\sin\phi_2 = \frac{3\pi}{4}, \quad \phi_2 = \sin^{-1}\left(\tfrac{1}{2}\right) = 30^\circ$$

$$\Rightarrow \frac{3\pi}{2}\sin\phi_3 = \frac{5\pi}{4}, \quad \phi_3 = \sin^{-1}\left(\tfrac{5}{6}\right) = 56.4^\circ$$

beamwidth along $\phi = 0$ is $9.6^\circ \times 2 = 19.2^\circ$; beamwidth along $\phi = 41.8^\circ$ is $(56.4^\circ - 30^\circ) = 26.4^\circ$



7.18 (a) $$\Delta = \frac{\lambda}{20\lambda\cos 30^\circ} = 3.3080^\circ$$

(b) $F(\theta, \phi = 90^\circ) = 2|\cos(20\pi\sin\theta)|$

at $\theta = 30^\circ$, $F(30^\circ, 90^\circ) = 2|\cos(10\pi)| = 2$ (a maximum)

null near $\theta = 30^\circ$ occurs at $\cos(20\pi\sin\theta) = \cos(10\pi \pm \frac{1}{2}\pi) = 0$

$20\pi\sin\theta_1 = 10.5\pi \longrightarrow \theta_1 = 31.6682^\circ$

$20\pi\sin\theta_2 = 9.5\pi \longrightarrow \theta_2 = 28.3594^\circ$

$\Delta = \theta_1 - \theta_2 = 3.3088^\circ$

7.19 $\langle \underline{S} \rangle = \frac{1}{2}\mathrm{Re}\{\bar{E} \times \bar{H}^*\} = \frac{\eta}{8\pi^2 r^2}(kbI_0)^2 \sin^2\phi \cos^2(\frac{\pi}{2}\cos\theta)$

$D(\theta,\phi) = \langle \underline{S} \rangle (4\pi r^2)/P_t = \frac{\eta}{8\pi^2 r^2}(kbI_0)^2 \sin^2\phi\cos^2(\frac{\pi}{2}\cos\theta) \times 4\pi r^2 / [\frac{\eta}{8\pi}(kbI_0)^2]$

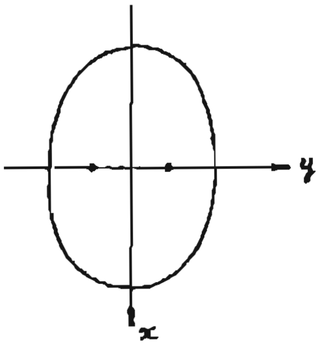
$= 4\sin^2\phi\cos^2(\frac{\pi}{2}\cos\theta)$

$D = D(90^\circ, 90^\circ) = 4$

7.20 $d = \lambda/4$, $kd = \pi/2$; $\theta = 90^\circ$, $\cos\gamma = \sin\phi$; $N = 2$; $\psi = 0$

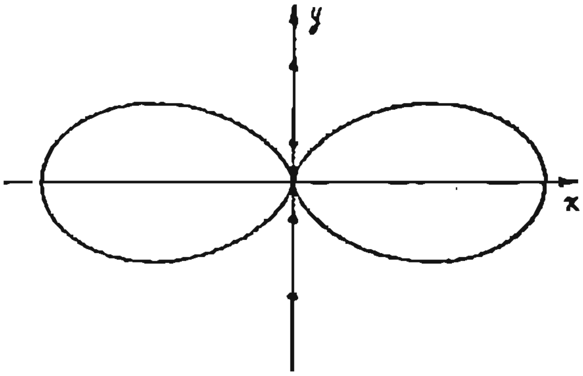
$|F(\phi)| = |\cos(\frac{\pi}{4}\sin\phi)|$

ϕ	0°	±10°	±20°	±30°	±40°	±50°	±60°	±70°	±80°	±90°
$\lvert F(\phi)\rvert$	1	0.99	0.96	0.92	0.88	0.82	0.78	0.74	0.72	0.71

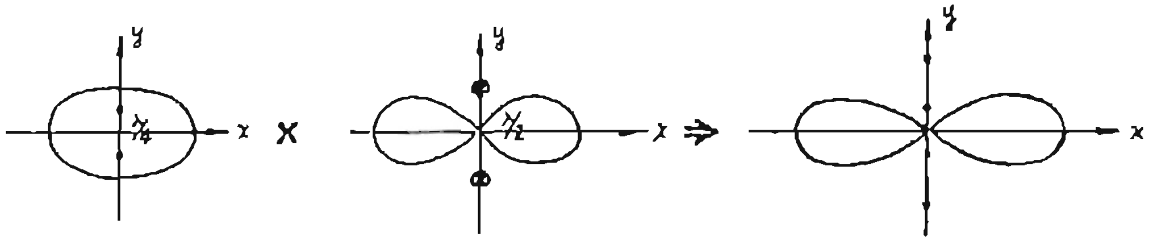


7.21 $d = \lambda/4$, $kd = \frac{\pi}{2}$; $\theta = 90°$, $\cos\gamma = \sin\phi$; $N = 4$; $\psi = 0$

(a) $|F(\phi)| = \left|\frac{\sin 4(\frac{\pi}{4}\sin\phi)}{\sin(\frac{\pi}{4}\sin\phi)}\right| = \left|\frac{\sin(\pi\sin\phi)}{\sin(\frac{\pi}{4}\sin\phi)}\right|$



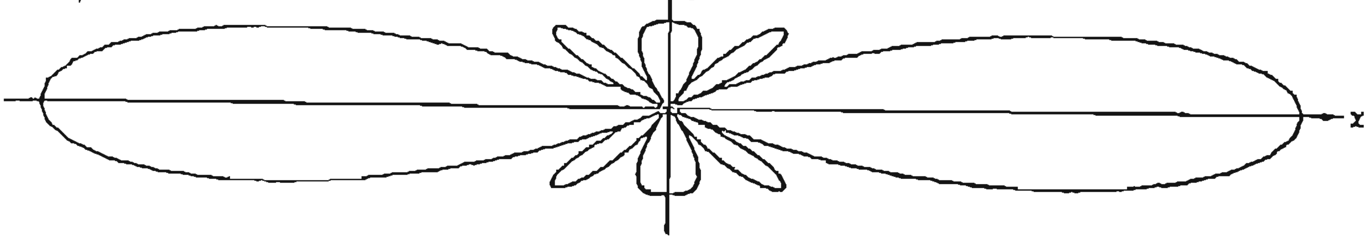
(b)



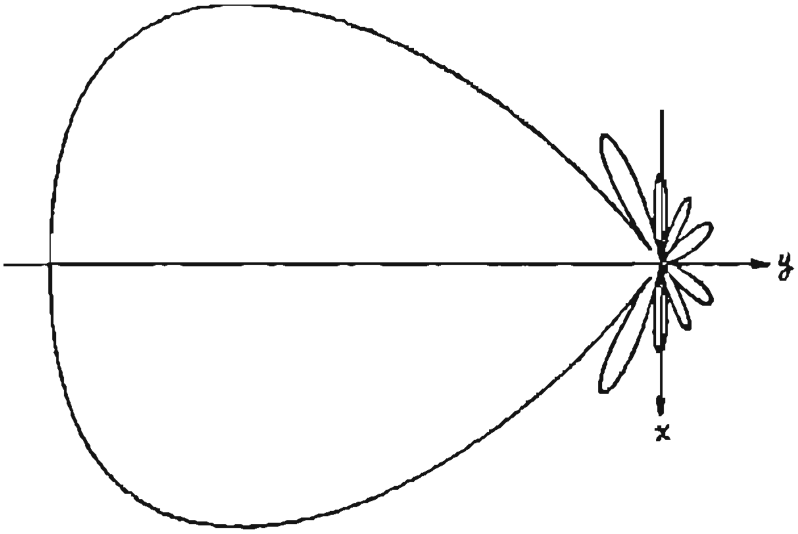
7.22 $d = \lambda/4$, $kd = \pi/2$; $\theta = 90°$, $\cos\gamma = \sin\phi$; $N = 10$

$|F(\phi,\psi)| = \left|\frac{\sin 10(\frac{\pi}{4}\sin\phi + \frac{\psi}{2})}{\sin(\frac{\pi}{4}\sin\phi + \frac{\psi}{2})}\right| = \left|\frac{\sin(2.5\pi\sin\phi + 5\psi)}{\sin(0.25\pi\sin\phi + 0.5\psi)}\right|$

(i) $\psi = 0°$



(ii) $\psi = 90°$



7.23 (a) $\psi = -\pi/2$.

(b) $E_\theta = \dfrac{1000}{r} e^{-j\beta r} \sin\theta \, \dfrac{\sin\left[\frac{3}{2}\pi(\sin\theta\sin\phi - 1)\right]}{\sin\left[\frac{1}{4}\pi(\sin\theta\sin\phi - 1)\right]}$

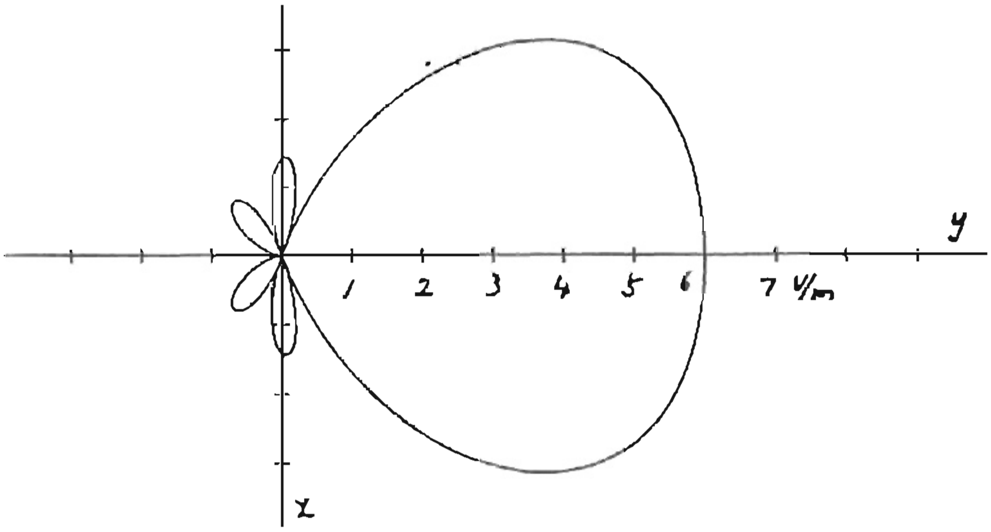
at A. $\phi = \pi/2$, $\theta = \pi/2$, $r = 1000$, $|E_\theta| = 6$ V/m

at B, $\phi = 0$, $\theta = \pi/2$, $r = 1000$, $|E_\theta| = 1.414$ V/m

at C, $\phi = -\pi/2$, $\theta = \pi/2$, $r = 1000$, $|E_\theta| = 0$

at D, $\phi = -\pi$, $\theta = \pi/2$, $r = 1000$, $|E_\theta| = 1.414$ V/m

(c) $\theta = \pi/2$, $|E_\theta| = \dfrac{1000}{r} \times \left| \dfrac{\sin\left[\frac{3}{2}\pi(\sin\phi - 1)\right]}{\sin\left[\frac{1}{4}\pi(\sin\phi - 1)\right]} \right|$



(d) $\phi = 0$, $|E_\theta| = \dfrac{1000}{r} \sqrt{2} \, |\sin\theta|$

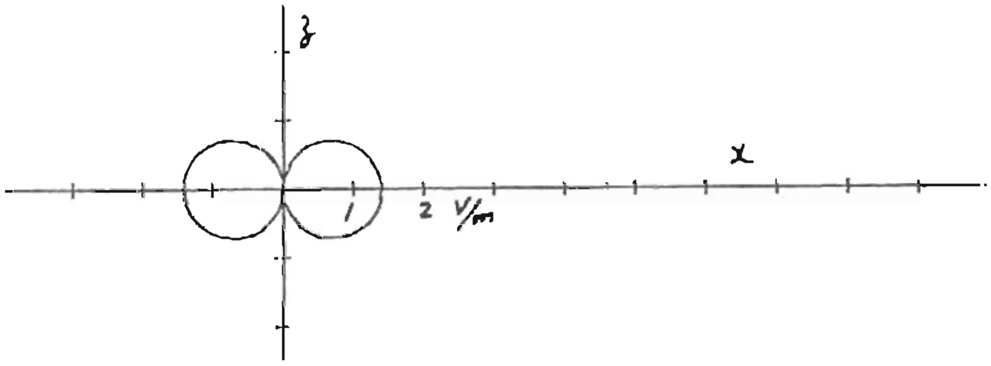


Figure 8.17 shows a typical arrangement of a liquid-crystal display.* It is operated in the so-called "distortion-of-aligned-phases" or DAP mode. Figure 8.17a shows the normal state of the crystal before activation. The light entering the crystal is polarized and then transmitted through the crystal with no alteration in polarization. The second polaroid absorbs all the light, and no light is transmitted. In its activated state, the crystal changes the polarization of the transmitted light, which propagates through the second polaroid and becomes visible.

Problems

8.1 The derivation of (8.5) only considers the electric field. Why is the magnetic field neglected? **Hint:** Compare the magnitude of E_r with ηH_r near the sphere, or the stored electric-energy density $(1/2)\epsilon|\mathbf{E}|^2$ with the stored magnetic-energy density $(1/2)\mu|\mathbf{H}|^2$.

8.2 Why is the rising or setting sun red?

8.3 The smoke emitted from engines of boats contains fine particles. Against a dark background the smoke looks blue but against a bright background it looks yellow. Why?

8.4 Explain the appearance of shafts of sunlight through breaks in a cloud-covered sky.

8.5 Show that

$$I_1 = \int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

Hint:

$$I_1^2 = \int_{-\infty}^{\infty} e^{-x^2}\,dx \cdot \int_{-\infty}^{\infty} e^{-y^2}\,dy$$

Then, transform x-y coordinates into cylindrical coordinates to perform the exact integration.

8.6 Show that

$$I_2 = \int_{-\infty}^{\infty} \exp(-p^2x^2 + qx)\,dx = \frac{\sqrt{\pi}}{p}\exp\left(\frac{q^2}{4p^2}\right)$$

Hint:

$$-p^2x^2 + qx = -\left(px - \frac{q}{2p}\right)^2 + \frac{q^2}{4p^2}$$

Then, use the result obtained in problem 8.5 after transforming the integration variable from x to $px - q/2p$.

8.7 Assume that on earth a microwave beam of 10 GHz is radiated by a 20-meter-diameter disk antenna aimed at the moon. Estimate the size of the microwave beam on the moon.

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*See R. W. Gurtler and C. Maze, "Liquid Crystal Displays," *IEEE Spectrum*, November 1972, p. 25.

8.8 A
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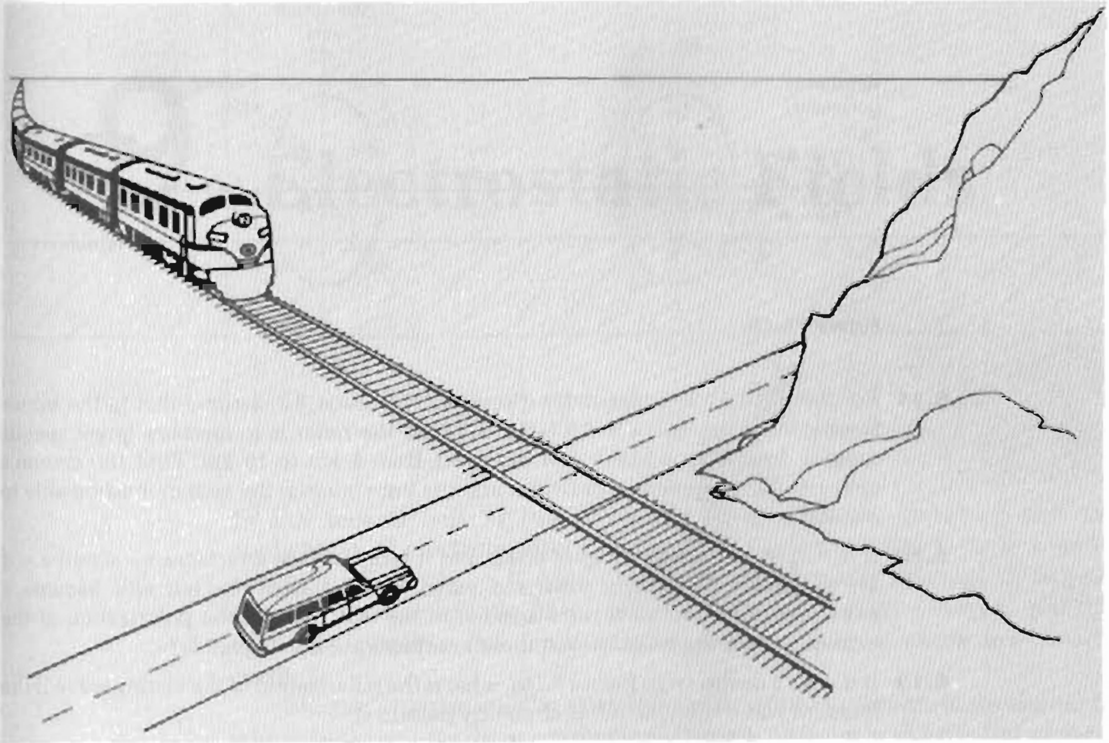


8.8 A person leaving his home by train mails a letter home every day. Suppose that the train travels 200 miles per day and that the mail moves at a speed of 200 miles per day. How frequently do his letters arrive home? Try to solve this problem by simple reasoning, not by substituting numbers in some formula.

8.9 On a foggy day, the driver of an automobile stopped at a railway crossing because he heard a whistle from a moving train. The sound of the whistle came from his left. A few seconds later he heard the echo, and the pitch of the first sound was lower than that of the echo. Assume that the echo was due to reflection from a nearby mountain close to the track. If you were the driver, would you cross the track—that is, could you tell whether the train was approaching or leaving you? (See Figure P8.9.)

8.10 A Doppler radar sends a signal at 8.800 GHz, and the receiver displays a frequency spectrum of returned signals as shown in Figure P8.10. What can you say about the speed of the target(s)?

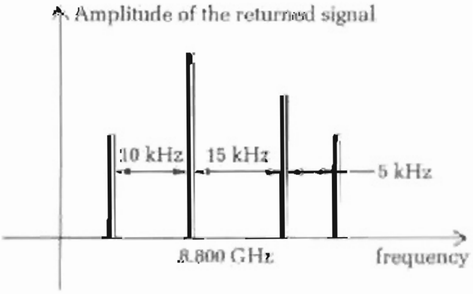


Figure P8.10

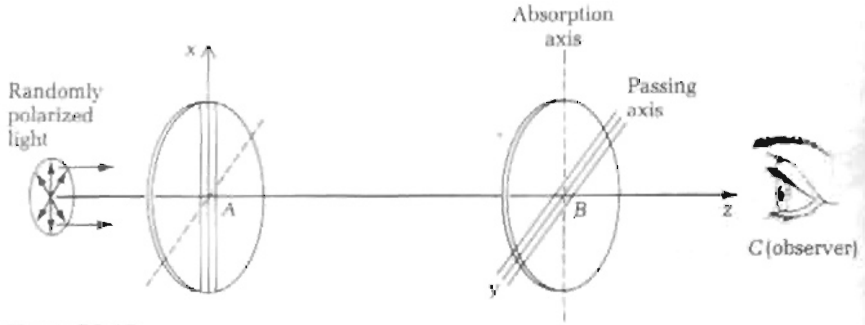


Figure P8.15

8.11 For the FM-CW Doppler radar discussed in Section 8.4, assume that f_0, the upper frequency of the radar, is 8.8 GHz. Suppose the radar is to measure target speeds ranging from 0 to 3 Mach and distances from 1 km to 10 km. Find the system's approximate frequency bandwidth and the time interval the system must be able to resolve.

8.12 If $d = \lambda_0/4$, as shown in Figure 8.16a, and if reflections at interfaces $z = 0$ and $z = d$ are negligible, a linearly polarized wave incident from the left will become a circularly polarized wave, as discussed in the text. What is the polarization of the exiting wave if the reflections at these interfaces are not negligible?

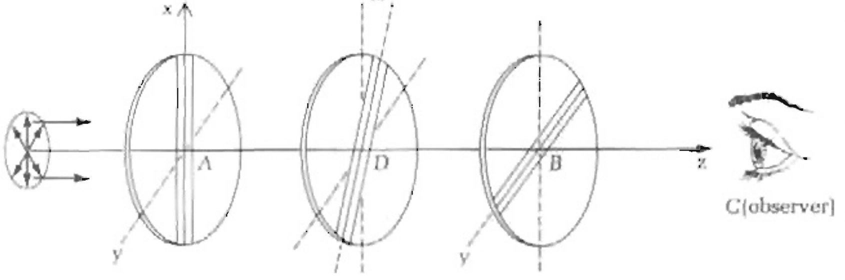
8.13 If $d = \lambda_0/2$ as shown in Figure 8.16a, what is the polarization of the exiting wave if the incident wave from the left is circularly polarized?

8.14 For a quartz crystal, $\epsilon_x = 2.41\,\epsilon_0$, and $\epsilon = 2.38\epsilon_0$. Find the minimum thickness of a quartz quarter-wave plate for a light having $\lambda = 6500$ Å.

8.15 In Figure P8.15 the Polaroid film at A is oriented such that it passes light polarized in the $\hat{x}$ direction and absorbs light polarized in the $\hat{y}$ direction. The film at B passes y-polarized light and absorbs x-polarized light. A randomly polarized light source, such as a flashlight, sheds light from the left along z. Can an observer at C see the light? Explain.

8.16 Consider the arrangement shown in Figure P8.16. This figure differs from Figure P8.15 only in the placement of a third Polaroid film at D between A and B. The absorption axis of the third film is 45° from either the x or the y axis. Now, can the observer at C see the light? (If you do not believe in your answer, do an experiment with three pairs of polarized sunglasses and see for yourself.)

Figure P8.16



CHAPTER 8

8.1 From (8.1), near the sphere, $|\underline{H}_\phi| \sim \frac{Ak\sin\theta}{\eta r^2}$, $|\underline{E}_r| \sim \frac{2A\cos\theta}{r^3}$, $|E_\theta| \sim \frac{A\sin\theta}{r^3}$

$\therefore \frac{\eta|\underline{H}_\phi|}{|\underline{E}|} \sim kr \ll 1$ and $\frac{U_H}{U_E} \sim \frac{\frac{\mu}{2}|\underline{H}|^2}{\frac{\epsilon}{2}|\underline{E}|^2} = \left|\frac{\eta H}{E}\right|^2 \sim (kr)^2 \ll 1$

8.2 Since high frequency light (blue) suffers higher scattering loss than low frequency light (red).

8.3 When background is dark, one sees light scattered by the smoke particles. Blue light is scattered more strongly than red light.
Against a bright background, one sees light passing through the smoke.
The blue light gets scattered and red and yellow light suffer less scattering.

8.4 the scattering of water and dust molecules

8.5 $I_1^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}dxdy = \int_0^{\infty}\int_0^{2\pi} e^{-\rho^2}\rho d\rho d\phi = 2\pi\int_0^{\infty}\rho e^{-\rho^2}d\rho = 2\pi\cdot\frac{1}{2}\int_0^{\infty}e^{-x}dx = \pi$

$\therefore I_1 = \int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$

8.6 $I_2 = \int_{-\infty}^{\infty} e^{(-p^2x^2+qx)}dx = \int_{-\infty}^{\infty} e^{-(px-\frac{q}{2p})^2}\cdot e^{\frac{q^2}{4p^2}}dx = e^{\frac{q^2}{4p^2}}\cdot\frac{1}{p}\int_{-\infty}^{\infty}e^{-u^2}du = \frac{\sqrt{\pi}}{p}e^{\frac{q^2}{4p^2}}$

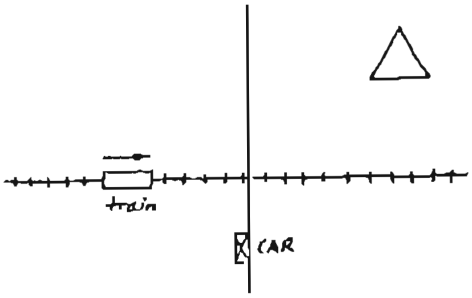
8.7 $f = 10\,GHz \Rightarrow \lambda = \frac{3\times10^8}{10\times10^9} = 0.03^m$; $W = \frac{20}{2} = 10^m$

$z_f = \frac{kW^2}{2} = \frac{2\pi}{\lambda}\frac{W^2}{2} = \frac{\pi}{0.03}\times(10)^2 = 1.047\times10^4$; $z = 3.8\times10^8 m$ = distance between earth and moon.

$\therefore$ Width $= Wz/z_f = 10\times3.8\times10^8/1.047\times10^4 = 3.63\times10^5 \propto 363\ km$

8.8 every other day

8.9



$f_1 = f + \Delta f$

$f_2 = f + 2\Delta f$

f_1 = frequency of the first sound

f_2 = frequency of the echo

$\therefore$ train is moving toward the intersection.

8.10 There are several targets:

Target #1 : $V_1 = \frac{\Delta f_1}{f_0}\cdot\frac{c}{2} = \frac{-10^4\times3\times10^8}{8.8\times10^9\times2} = -170.5\ m/s$, away from the receiver.

Target #2 : $V_2 = \frac{\Delta f_2}{f_0}\cdot\frac{c}{2} = \frac{15\times10^3\times3\times10^8}{8.8\times10^9\times2} = 255.7\ m/s$, toward the receiver

Target #3 : $V_3 = \frac{\Delta f_3}{f_0}\cdot\frac{c}{2} = \frac{20\times10^3\times3\times10^8}{8.8\times10^9\times2} = 341\ m/s$, toward the receiver.

8.11 $\Delta f = f_o(2u/c) = \frac{8.8\times10^9\times2\times993}{3\times10^8} = 58256\ Hz$

$2r/c = 2\times10^3/3\times10^8 = 6.7\times10^{-6}\ s = 6.7\ \mu s$

$\therefore$ Bandwidth = 59 KHz ; Time resolution : 1 km ⟷ 6.7 μs.

8.12 Elliptically polarized wave

8.13 Circular polarized but opposite hand.

8.14 $d = \frac{\pi/2}{k_e - k_o} = \frac{\pi/2}{2\pi/\lambda(\sqrt{\epsilon_x} - \sqrt{\epsilon})} = \frac{\lambda}{4(\sqrt{\epsilon_x} - \sqrt{\epsilon})} = \frac{6500\text{Å}}{4(\sqrt{2.41} - \sqrt{2.38})} = 16.76\ \mu m$

8.15 NO.

8.16 YES.

Problems

9.1 Consider the dipole arrangement shown in Figure 9.2a. Let $q = 1.6 \times 10^{-19}$ C. Find Φ at:

(a) $x = 0.1, y = 0.1, z = 0.1$
(b) $x = 1, y = 1, z = 1$

Use the exact formula (9.14) first. Then use the approximate formula (9.17), and find the accuracy of the latter. The medium is air.

9.2 Three point charges are located on the x axis with $q_1 = q$ at $x = 0$, $q_2 = 2q$ at $x = 1$, and $q_3 = -3q$ at $x = 2$. Find the position(s) on the x axis where Φ is equal to zero.

9.3 Four point charges are located on the corners of a rectangle, as shown in Figure P9.3. Find the planes on which the potential is equal to zero. Sketch these planes.

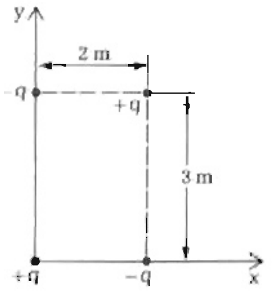


Figure P9.3

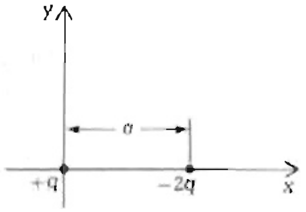


Figure P9.4

9.4 Two point charges are separated by a meters in air, as shown in Figure P9.4.

(a) Find the potential function $\Phi(x, y, z)$.
(b) Calculate Φ at $x = 100a$, $y = 100a$, $z = 0$.
(c) Show that, for distances much greater than a away from these charges, the potential is approximately given by

$$\Phi \approx \frac{-q}{4\pi\epsilon_0} \frac{1}{(x^2 + y^2 + z^2)^{1/2}}$$

where $(x^2 + y^2 + z^2)^{1/2} >> a$. Use this approximate formula to calculate $\Phi(100a, 100a, 0)$, and compare it with the result obtained in (b).

9.5 Find the **E** field in air due to a point charge of $10^6 q_e$ ($q_e = -1.6 \times 10^{-19}$ C). Sketch a diagram similar to Figure 9.4.

9.6 For the charge distribution given in Problem 9.2, calculate E_x at (a) $x = -1$, (b) $x = 0.5$, (c) $x = 2.5$, and (d) $x = 3$.

9.7 Apply $\mathbf{E} = -\nabla\Phi$ to (9.14) to find the **E** field located at the origin and produced by two charges $+q$ and $-q$ located at $(0, 0, h)$ and $(0, 0, -h)$, respectively, as shown in Figure 9.2a, where $h = 1$ cm. Show that $\mathbf{E} = -\hat{z}(q/2\pi\epsilon h^2)$.

9.8 Solve the same problem as in 9.7, but use (9.19c).

9.9 Sketch the direction of the **E** field located at the center of a square shown in Figure P9.9. The **E** field is produced by four charges at four corners of the square $OABC$, these four charges carry q, $-q$, $-q$, and q coulombs, respectively.

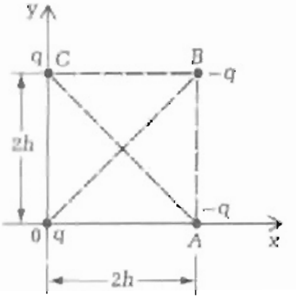


Figure P9.9

9.10 A line charge $2h$ meters long is located along the z axis as shown in Figure 9.6a. The charge density is ρ_ℓ coulombs per meter.

(a) Calculate the electric field at $\rho = 0.1h$, $\phi = 0$, and $z = 0$ using the exact formula (9.21).

(b) Calculate the electric field at the same point using the assumption that the line is infinitely long.

(c) Find the percentage error of the value obtained in (b) as compared with the exact value.

9.11 For the same line-charge described in Problem 9.10,

(a) Calculate the electric field at $\rho = 20h$, $\phi = 0$, $z = 0$ using the exact formula.

(b) Do the same using the assumption that the line is a point charge at the origin.

(c) Find the percentage error of value obtained in (b).

9.12 A plane charge of ρ_s coulombs per square meter is located on the $x = 0$ plane, and another plane of $-\rho_s$ coulombs per square meter is located on the $x = 1$ plane. Find the total electric-field in the region (a) $x > 1$, (b) $1 > x > 0$, and (c) $x < 0$.

9.13 Consider the problem discussed in Example 9.11. Assuming that everything is the same except for the fact that the total charge on the conducting shell is now equal to zero, calculate **E** everywhere, and sketch E_r versus r similar to the sketch shown in Figure 9.15.

9.14 A charge distribution of the following form is set up in air (spherical coordinates):

$$\rho_v = \begin{cases} 0 & 0 < r < a \\ 10^{-6} & a < r < b \\ 0 & b < r \end{cases}$$

(a) Find the **D** field for $0 < r < a$.

(b) Find **D** for $a < r < b$.

(c) Find **D** for $b < r$.

9.15 A charge distribution of the following form is set up in air:

$$\rho_v = 10^{-6} \cdot e^{-r} \text{ coulombs per cubic meter}$$

Use Gauss' law to find the **E** field everywhere. **Hint:** To find the total charge in a Gaussian surface, you must do the integration because the charge is not uniformly distributed. However, symmetry still exists with respect to ϕ and θ.

9.16 Electric charges are distributed uniformly in the region $0.1 < x < +0.1$ with density $\rho_v = 10^{-6}$ C/m^3. Elsewhere, the density is equal to zero. Find the **E** field everywhere.

Plot E_x versus x. Find the potential difference $V_x - V_0$ for a point x with respect to the origin.

9.17 Find the potential difference $V_A - V_B$ for two points A and B located at $r = 0$ and $r = 1$ in the **E** field obtained in Problem 9.15.

9.18 The solution for the electric field of an oscillating Hertzian dipole with angular frequency ω is given in (7.14) as follows:

$$\mathrm{E} = \sqrt{\frac{\mu}{\epsilon}}\frac{jkI\,\Delta z\,e^{-jkr}}{4\pi r}\left\{\hat{r}\left[\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]2\cos\theta+\hat{\theta}\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]\sin\theta\right\}$$

Derive the solution (9.20) for a static dipole by setting $\omega = 0$. Notice that $k = \omega(\mu\epsilon)^{1/2}$ and $I\,\Delta z = \partial p/\partial t = j\omega p$.

9.19 In the electric field $\mathbf{E} = 3\hat{x} + 4\hat{y} - 5\hat{z}$, find $V_A - V_B$ if A is located at (1, 1, 2) and B is at the origin. Does the difference depend on the path of the integration?

9.20 Consider the spherical-shell problem shown in Figure 9.14. Find the potential $\Phi(r)$ at (a) $r = c$; (b) $b < r < c$; (c) $b > r > a$, and (d) $r = a$. Assume $\Phi = 0$ at infinity. Plot $\Phi(r)$ versus r.

9.21 Repeat the preceding problem for the case in which the total charge on the conducting shell is equal to zero while all other conditions remain unchanged. You may want to use the result obtained in Problem 9.13.

9.22 Consider the coaxial line shown in Figure P9.22. The inner conductor is a solid conducting cylinder with a radius equal to 0.1 m. The outer conductor has an inner radius equal to 0.4 m and an outer radius equal to 0.5 m. The medium between the inner and the outer conductor is air. The inner conductor carries a net charge of $-3\epsilon_0$ C/m and the outer conductor carries a net charge of $-15\epsilon_0$ C/m. The symbol ϵ_0 used here represents a constant equal to 8.854×10^{-12}.

(a) Find E_ρ in the region 0.1 m $< \rho <$ 0.4 m.
(b) Find E_ρ in the region 0.4 m $< \rho <$ 0.5 m.
(c) Find E_ρ in the region $\rho >$ 0.5 m.
(d) Find Φ at $\rho = 0.2$ m, knowing that $\Phi = 0$ at $\rho = 1$ m.
(e) Sketch E_ρ as function of ρ for $0<\rho<1$ m. Mark the scale for E_ρ and ρ.

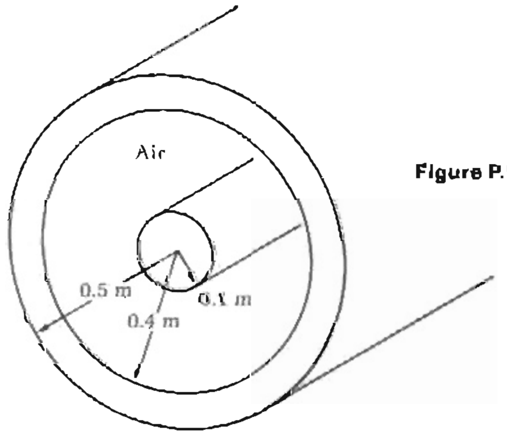


Figure P.9.22

vith respect to

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with angular

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hat $k = \omega(\mu\epsilon)^{1/2}$

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9.23 Model the dome of a Van de Graaff generator as a conducting sphere. The dome is charged to hold the maximum amount of electric charge Q_m before the air surrounding the dome breaks down. Use the following data:

radius of the dome = 0.11 m,
breakdown E of air = 3×10^6 V/m.

(a) Calculate the maximum Q_m accumulated on the dome just before the breakdown.

(b) Calculate the voltage of the dome in reference to the potential at infinity just before breakdown occurs.

(c) When the dome is charged with the maximum charge Q_m, a person uses a conducting rod to discharge the electricity. Assume that the discharge takes 0.01 seconds to complete, how strong is the discharging current (on the average)?

CHAPTER 9

9.1 EXACT: (a) $\Phi(0.1,0.1,0.1)=\frac{1.6\times10^{-19}\times36\pi\times10^{9}}{4\pi}\left\{\left[(0.1)^2+(0.1)^2+(0.09)^2\right]^{-\frac{1}{2}}-\left[(0.1)^2+(0.1)^2+(0.11)^2\right]^{-\frac{1}{2}}\right\}$

$=5.5302357\times10^{-10}$ V

(b) $\Phi(1,1,1)=\frac{1.6\times10^{-19}\times36\pi\times10^{9}}{4\pi}\left\{\left[1^2+1^2+(0.99)^2\right]^{-\frac{1}{2}}-\left[1^2+1^2+(1.01)^2\right]^{-\frac{1}{2}}\right\}$

$=5.542439\times10^{-12}$ V

Approximate: (a) $r=\left[(0.1)^2+(0.1)^2+(0.1)^2\right]^{\frac{1}{2}}=\sqrt{0.03}$, $\theta=\cos^{-1}\left(\frac{0.1}{\sqrt{0.03}}\right)=54.7^\circ$

$\Phi(\sqrt{0.03},54.7^\circ)=\frac{1.6\times10^{-19}\times0.02\times\cos(54.7^\circ)}{4\pi\times(0.03)}\times36\pi\times10^{9}=5.5425626\times10^{-10}$ V

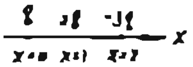
(Error ≐ 0.223 %)

(b) $r=\sqrt{3}$, $\theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)=54.7^\circ$

$\Phi(\sqrt{3},54.7^\circ)=\frac{1.6\times10^{-19}\times0.02\times\cos(54.7^\circ)}{4\pi\times3}\times36\pi\times10^{9}=5.5425626\times10^{-12}$ V

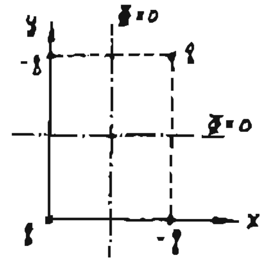
(Error ≐ 0.0022%)

9.2 $\Phi=\frac{q}{4\pi\varepsilon}\left[\frac{1}{|x|}+\frac{2}{|x-1|}-\frac{3}{|x-2|}\right]=0\Rightarrow\frac{1}{|x|}+\frac{2}{|x-1|}-\frac{3}{|x-2|}=0$



the only solution is for $1<x<2$. $\therefore\frac{1}{x}+\frac{2}{x-1}+\frac{3}{x-2}=0\Rightarrow x=\frac{5+\sqrt{13}}{6}=1.43$

9.3



$x=1$ m plane

$y=1.5$ m plane

9.4 (a) $\Phi(x,y,z)=\frac{q}{4\pi\varepsilon}\left\{(x^2+y^2+z^2]^{-\frac{1}{2}}-2\left[(x-a)^2+y^2+z^2\right]^{-\frac{1}{2}}\right\}$

(b) $\Phi(100a,100a,0)=\frac{q}{4\pi\varepsilon}\left\{\left[(100a)^2+(100a)^2\right]^{-\frac{1}{2}}-2\left[(99a)^2+(100a)^2\right]^{-\frac{1}{2}}\right\}=\frac{-q}{4\pi\varepsilon a}(0.00714)$

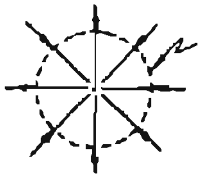
(c) Since $(x^2+y^2+z^2)^{1/2}\gg a$, $\left[(x-a)^2+y^2+z^2\right]^{1/2}\sim(x^2+y^2+z^2)^{1/2}$

From (a), $\Phi(x,y,z)\sim\frac{q}{4\pi\varepsilon}\left\{\left[x^2+y^2+z^2\right]^{-\frac{1}{2}}-2\left[x^2+y^2+z^2\right]^{-\frac{1}{2}}\right\}=-\frac{q}{4\pi\varepsilon}(x^2+y^2+z^2)^{-\frac{1}{2}}$

$\Phi(100a,100a,0)\sim\frac{-q}{4\pi\varepsilon a}\frac{1}{\sqrt{2}\cdot100}=-\frac{q}{4\pi\varepsilon a}(0.00707)$

9.5 $\bar{E}=\frac{-1.6\times10^{-10}}{4\pi r^2}\times10^{6}\times36\pi\times10^{9}\,\hat{r}$

$=-\frac{0.00144}{r^2}\hat{r}$



$|\bar{E}|=14.4$ V/m on $|\bar{r}|=1$ cm

9.6 (a) $E_x=\frac{q}{4\pi\varepsilon}\left[-\frac{1}{1}-\frac{2}{2^2}-\frac{(-3)}{3^2}\right]=\frac{q}{4\pi\varepsilon}\left(-\frac{7}{6}\right)$

(b) $E_x=\frac{q}{4\pi\varepsilon}\left[\frac{1}{(0.5)^2}-\frac{2}{(0.5)^2}+\frac{3}{(1.5)^2}\right]=\frac{q}{4\pi\varepsilon}(-2.67)$

(c) $E_x=\frac{q}{4\pi\varepsilon}\left[\frac{1}{(2.5)^2}+\frac{2}{(1.5)^2}-\frac{3}{(0.5)^2}\right]=\frac{q}{4\pi\varepsilon}(-10.95)$

(d) $E_x=\frac{q}{4\pi\varepsilon}\left[\frac{1}{3^2}+\frac{2}{2^2}-\frac{3}{1}\right]=\frac{q}{4\pi\varepsilon}(-2.389)$

9.7 $\Phi(x,y,z)=\frac{q}{4\pi\varepsilon}\left[\frac{1}{\sqrt{x^2+y^2+(z-h)^2}}-\frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right]$

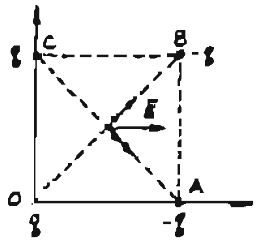
$\bar{E}=-\nabla\Phi=\frac{q}{4\pi\varepsilon}\left\{\frac{\hat{x}x+\hat{y}y+\hat{z}(z-h)}{\left[x^2+y^2+(z-h)^2\right]^{3/2}}-\frac{\hat{x}x+\hat{y}y+\hat{z}(z+h)}{\left[x^2+y^2+(z+h)^2\right]^{3/2}}\right\}\Rightarrow\bar{E}(0,0,0)=\frac{q}{4\pi\varepsilon}\left\{\frac{\hat{z}}{h^3}(-h)-\frac{\hat{z}}{h^3}(h)\right\}=-\hat{z}\frac{q}{2\pi\varepsilon h^2}$

9.8 $\bar{E}=-\nabla\left\{\frac{q}{4\pi\varepsilon}\frac{1}{\left[x^2+y^2+(z-h)^2\right]^{1/2}}-\frac{q}{4\pi\varepsilon}\frac{1}{\left[x^2+y^2+(z+h)^2\right]^{1/2}}\right\}=-\nabla\left\{\frac{q}{4\pi\varepsilon}\frac{1}{\left[x^2+y^2+(z-h)^2\right]^{1/2}}\right\}-\nabla\left\{\frac{q}{4\pi\varepsilon}\frac{1}{\left[x^2+y^2+(z+h)^2\right]^{1/2}}\right\}$

$=\frac{q}{4\pi\varepsilon}\left\{\frac{\hat{x}x+\hat{y}y+\hat{z}(z-h)}{\left[x^2+y^2+(z-h)^2\right]^{3/2}}-\frac{\hat{x}x+\hat{y}y+\hat{z}(z+h)}{\left[x^2+y^2+(z+h)^2\right]^{3/2}}\right\}$

$\therefore\bar{E}(0,0,0)=-\hat{z}\frac{q}{2\pi\varepsilon h^2}$

9.9



9.10 (a) $\rho = 0.1h,\ \alpha_0 = \tan^{-1}(h/0.1h) = 84.9°$

$$\bar{E} = \hat{\rho}\frac{\rho_\ell}{2\pi\varepsilon(0.1h)}\sin(84.3°) = \hat{\rho}\frac{\rho_\ell}{2\pi\varepsilon h}(9.95)$$

(b) For infinitely long line, $E = \hat{\rho}\frac{\rho_\ell}{2\pi\varepsilon(0.1h)} = \hat{\rho}\frac{\rho_\ell}{2\pi\varepsilon h}(10)$

(c) $\text{Error} = \frac{10-9.95}{9.95}\times 100\% = 0.503\%$

9.11 $\rho = 20h,\ \alpha_0 = \tan^{-1}(h/20h) = 2.86°$

(a) $\bar{E} = \hat{\rho}\frac{\rho_\ell}{2\pi\varepsilon(20h)}\sin(2.86°) = \hat{\rho}\frac{\rho_\ell}{40\pi\varepsilon h}(0.0499)$

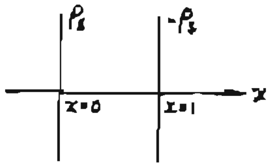
(b) point charge approximation, $\bar{E} = \hat{\rho}\frac{\rho_\ell \times 2h}{4\pi\varepsilon(20h)^2} = \hat{\rho}\frac{\rho_\ell}{40\pi\varepsilon h}(0.05)$

(c) $\text{Error} = \frac{0.05-0.0499}{0.0499}\times 100\% = 0.2\%$

9.12 (a) $\bar{E} = 0$ for $x > 1$

(b) $\bar{E} = 2\left(\frac{\rho_s}{2\varepsilon}\right)\hat{x} = \hat{x}\left(\frac{\rho_s}{\varepsilon}\right)$ for $0 < x < 1$

(c) $\bar{E} = 0$ for $x < 0$

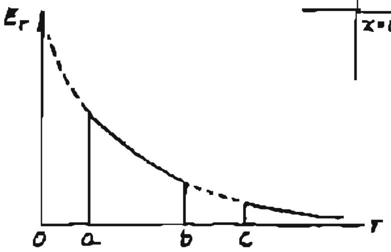


9.13 $\bar{E} = 0$ for $r < a$

$\bar{E} = \hat{r}\frac{q}{4\pi\varepsilon r^2}$ for $a < r < b$

$\bar{E} = 0$ for $b < r < c$

$\bar{E} = \hat{r}\frac{q}{4\pi\varepsilon r^2}$ for $r \geq c$



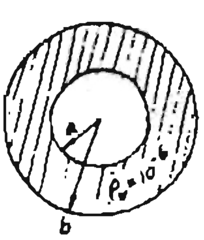
9.14 (a) for $0 < r < a$, $\bar{D} = 0$ (since no ρ_v enclosed)

(b) for $a < r < b$, $(4\pi r^2)D_r = \int_v \rho_v\,dv = \rho_v\frac{4\pi}{3}(r^3 - a^3)$

$\therefore\ \bar{D} = \hat{r}\frac{\rho_v}{3}\cdot\frac{(r^3-a^3)}{r^2}$, with $\rho_v = 10^{-6}$

(c) for $b < r$, $(4\pi r^2)D_r = \int_v \rho_v\,dv = \rho_v\frac{4\pi}{3}(b^3 - a^3)$

$\therefore\ \bar{D} = \hat{r}\frac{\rho_v}{3}\cdot\frac{(b^3-a^3)}{r^2}$, with $\rho_v = 10^{-6}$



9.15 Apply Gauss' law: $E_r(4\pi r^2) = \frac{1}{\varepsilon_0}\int_v \rho_v\,dv$

$$= \frac{1}{\varepsilon_0}\int_0^r\int_0^{2\pi}\int_0^{\pi}(10^{-6}e^{-r})r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$= 10^{-6}\cdot\frac{4\pi}{\varepsilon_0}\int_0^r r^2e^{-r}dr$$

$$\therefore\ E_r = \frac{10^{-6}}{\varepsilon_0 r^2}\int_0^r r^2e^{-r}dr = \frac{10^{-6}}{\varepsilon_0 r^2}\left[-e^{-r}(r^2+2r+2)\right]_0^r = \frac{10^{-6}}{\varepsilon_0 r^2}\left[2 - e^{-r}(r^2+2r+2)\right]$$

9.16 for $|x| < 0.1$, $\pi h^2(D_x) - \pi h^2(-D_x) = 2\pi h^2(D_x) = \rho_v\,\pi h^2(2x)$

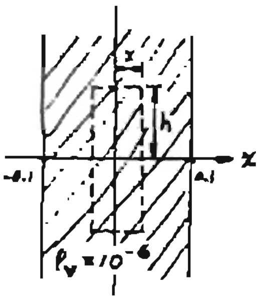
$\therefore\ D_x = \rho_v x \Rightarrow \bar{E} = \hat{x}\frac{\rho_v}{\varepsilon_0}x$, where $\rho_v = 10^{-6}$

for $x > 0.1$, $2\pi h^2(D_x) = \rho_v\,\pi h^2(0.2)$

$\therefore\ D_x = 0.1\rho_v \Rightarrow \bar{E} = \hat{x}\frac{\rho_v}{\varepsilon_0}(0.1)$, where $\rho_v = 10^{-6}$

for $x < -0.1$, $2\pi h^2(-D_x) = \rho_v\,\pi h^2(0.2)$

$\therefore\ D_x = -0.1\rho_v \Rightarrow \bar{E} = -\hat{x}\frac{\rho_v}{\varepsilon_0}(0.1)$, where $\rho_v = 10^{-6}$



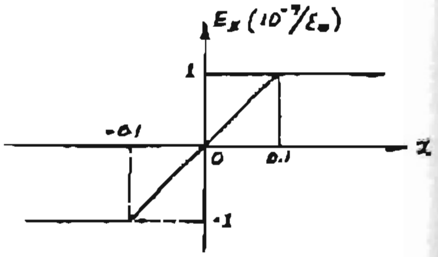
for $x<-0.1$, $V_{x0} = -\int_0^x E_x\,dx = -\int_0^{-0.1} \frac{\rho_v}{\varepsilon_0} x\,dx - \int_{-0.1}^{x} -\frac{(0.1)\rho_v}{\varepsilon_0}\,dx$

$$= -\frac{\rho_v}{\varepsilon_0}\left\{0.005 - 0.1x - 0.01\right\} = \frac{\rho_v}{\varepsilon_0}(0.1x + 0.005)$$

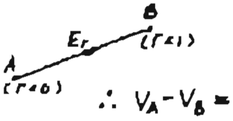
for $|x|<0.1$, $V_{x0} = -\int_0^x E_x\,dx = -\int_0^x \frac{\rho_v}{\varepsilon_0} x\,dx = -\frac{\rho_v}{\varepsilon_0}\frac{x^2}{2}$

for $x>0.1$, $V_{x0} = -\int_0^x E_x\,dx = -\int_0^{0.1} \frac{\rho_v}{\varepsilon_0} x\,dx - \int_{0.1}^{x} \frac{\rho_v}{\varepsilon_0}(0.1)\,dx$

$$= -\frac{\rho_v}{\varepsilon_0}\left\{0.005 + 0.1x - 0.01\right\} = \frac{\rho_v}{\varepsilon_0}(0.005 - 0.1x)$$



9.17 $V_A - V_B = -\int_B^A E_r\,dr = -\int_1^0 \frac{10^{-6}}{\varepsilon_0 r^2}\left[2 - e^{-r}(r^2+2r+2)\right]dr = \frac{10^{-6}}{\varepsilon_0}\left\{\int_0^1 \frac{2}{r^2}dr - \int_0^1 e^{-r}dr - 2\int_0^1\left(\frac{e^{-r}}{r} + \frac{e^{-r}}{r^2}\right)dr\right\}$



Since there is a singularity at $r=0$, take lower limit of the integrations to be δ with δ positive and $\delta \ll 1$.

$$\therefore V_A - V_B = \lim_{\delta\to 0}\frac{10^{-6}}{\varepsilon_0}\left\{\int_\delta^1 \frac{2}{r^2}dr - \int_\delta^1 e^{-r}dr - 2\int_\delta^1\left(\frac{e^{-r}}{r}+\frac{e^{-r}}{r^2}\right)dr\right\}$$, where $\int\left(\frac{e^{-r}}{r}+\frac{e^{-r}}{r^2}\right)dr = -\frac{e^{-r}}{r}$

$$= \lim_{\delta\to 0}\frac{10^{-6}}{\varepsilon_0}\left\{-2 + \frac{2}{\delta} + \frac{1}{e} - e^{-\delta} + \frac{2}{e} - \frac{2}{\delta}e^{-\delta}\right\} = \lim_{\delta\to 0}\frac{10^{-6}}{\varepsilon_0}\left\{3e^{-1} - 2 - e^{-\delta} + \frac{2}{\delta}(1-e^{-\delta})\right\}$$

$$= \frac{10^{-6}}{\varepsilon_0}\left\{3e^{-1} - 2 - 1 + \lim_{\delta\to 0}\frac{2(1-e^{-\delta})}{\delta}\right\} = \frac{10^{-6}}{\varepsilon_0}\left\{3e^{-1} - 1\right\}$$

9.18 $\underline{\bar{E}} = \sqrt{\frac{\mu}{\varepsilon}}\cdot\frac{jk(j\omega p)}{4\pi r}e^{-jkr}\left\{\hat{r}\left[\frac{1}{jkr} + \frac{1}{(jkr)^2}\right]2\cos\theta + \hat{\theta}\left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right]\sin\theta\right\}$

$$= \sqrt{\frac{\mu}{\varepsilon}}\cdot\frac{jk(j\omega p)}{4\pi r(jkr)^2}e^{-jkr}\left\{\hat{r}\left[jkr+1\right]2\cos\theta + \hat{\theta}\left[(jkr)^2 + jkr + 1\right]\sin\theta\right\}$$

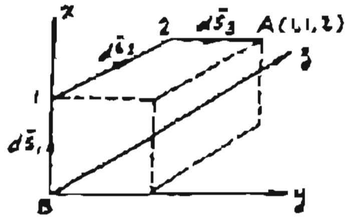
$$= \sqrt{\frac{\mu}{\varepsilon}}\cdot\frac{p\,e^{-jkr}}{4\pi r^3\sqrt{\mu\varepsilon}}\left\{\hat{r}\left[jkr+1\right]2\cos\theta + \hat{\theta}\left[(jkr)^2 + jkr + 1\right]\sin\theta\right\}$$

$$= \frac{p\,e^{-jkr}}{4\pi\varepsilon r^3}\left\{\hat{r}\left[jkr+1\right]2\cos\theta + \hat{\theta}\left[(jkr)^2 + jkr + 1\right]\sin\theta\right\} \xrightarrow[k\to 0]{\omega\to 0} \frac{p}{4\pi\varepsilon r^3}\left\{\hat{r}\,2\cos\theta + \hat{\theta}\sin\theta\right\}$$

9.19 $V_A - V_B = -\int_0^1 \bar{E}\cdot d\bar{s}_1 - \int_0^2 \bar{E}\cdot d\bar{s}_2 - \int_0^1 \bar{E}\cdot d\bar{s}_3$, where $\bar{E} = 3\hat{x} + 4\hat{y} - 5\hat{z}$

$$= -9\cdot 1 - (-5)\cdot 2 - 4\cdot 1 = 3\text{ V}$$

$\nabla\times\bar{E} \equiv 0 \Rightarrow V_A - V_B$ is independent of the path of the integration



9.20 (a) $\Phi(r=c) = -\int_\infty^c \frac{q-q'}{4\pi\varepsilon r^2}dr = \frac{q-q'}{4\pi\varepsilon c}$

(b) $\Phi(b<r<c) = \Phi(c) + 0 = \frac{q-q'}{4\pi\varepsilon c}$

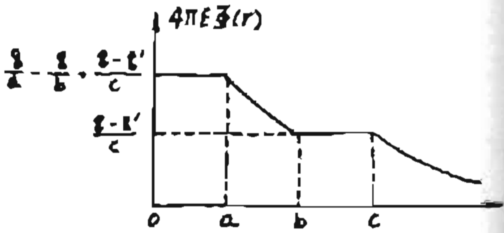
(c) $\Phi(a<r<b) = \Phi(b) - \int_b^r \frac{q}{4\pi\varepsilon r^2}dr = \frac{q-q'}{4\pi\varepsilon c} + \frac{q}{4\pi\varepsilon r} - \frac{q}{4\pi\varepsilon b}$

$$= \frac{1}{4\pi\varepsilon}\left(\frac{q}{r} - \frac{q}{b} + \frac{q}{c} - \frac{q'}{c}\right)$$

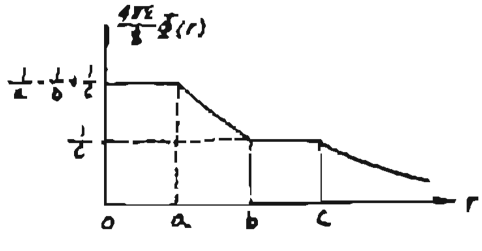
(d) $\Phi(r=a) = \Phi(b) - \int_b^a \frac{q}{4\pi\varepsilon r^2}dr = \frac{q-q'}{4\pi\varepsilon c} + \frac{q}{4\pi\varepsilon a} - \frac{q}{4\pi\varepsilon b}$

$$= \frac{1}{4\pi\varepsilon}\left(\frac{q}{a} - \frac{q}{b} + \frac{q}{c} - \frac{q'}{c}\right)$$

also, $\Phi(r>c) = -\int_\infty^r \frac{q-q'}{4\pi\varepsilon r^2}dr = \frac{q-q'}{4\pi\varepsilon r}$



9.21 Let $q'=0$ in Problem 9.20



9.22 (a) $E_\rho = \dfrac{-3}{2\pi\rho}$ $\quad 0.1 < \rho < 0.4$

(b) $E_\rho = 0$ $\quad 0.4 < \rho < 0.5$

(c) $E_\rho = \dfrac{-3-15}{2\pi\rho} = \dfrac{-9}{\pi\rho}$ for $\rho > 0.5$

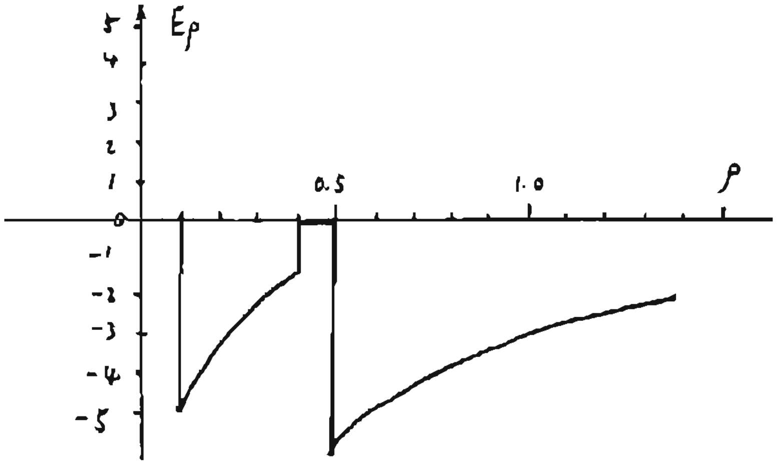
(d) $\Phi(0.2) = -\int_1^{0.2} E_\rho \, d\rho$

$$= -\int_1^{0.5} \frac{-9}{\pi\rho} d\rho - \int_{0.5}^{0.4} 0 \, d\rho - \int_{0.4}^{0.2} \frac{-3}{2\pi\rho} d\rho$$

$$= \frac{9}{\pi} \ln(0.5) + \frac{3}{2\pi} \ln(0.5)$$

$$= -2.32 \text{ V}$$

(e)



9.23 (a) $E_b = \dfrac{Q_m}{4\pi\varepsilon a^2}$ $\quad E_b = 3\times10^{6}$, $a = 0.11$, $\varepsilon = 8.854\times10^{-12}$,

$Q_m = 4.03\times10^{-6}$ Coul.

(b) $V = \dfrac{Q_m}{4\pi\varepsilon a} = 330$ kV

(c) $I = \dfrac{Q_m}{\Delta t} = 0.4$ mA

This electrostatic adhesive surface is widely used in desk-top calculator-driven curve tracers. A typical voltage used to charge the embedded conductors is 300 volts, and typical spacings between them are approximately 2 mm.*

Problems

10.1 A point charge of q coulombs is located at the origin (0, 0, 0), and a second point charge of q' coulombs is at (1, 0, 0). A small test-charge is placed at (3, 0, 0), and it is found that the total force on the test charge is equal to zero. Find q' in terms of q.

10.2 Two identical small balls are attached to weightless strings 15 cm long. Each ball carries 10^{-9} C of charge, and each has a mass of 1 g. They achieve an equilibrium state under the influence of electrostatic force and gravitational force, as shown in Figure P10.2. Find the angle α. **Hint:** α is small.

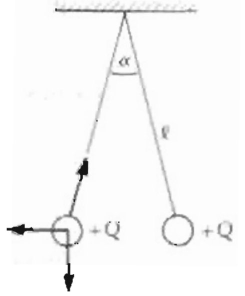


Figure P10.2

10.3 Consider a long line-charge with $\rho_\ell = 10^{-6}$ C/m. Find the force acting on a dust particle carrying -10^{-9} C, 1 m away from the line charge.

10.4 A line charge with $\rho_\ell = 10^{-6}$ C/m is located in air at $x = 1$, $y = 0$. A plane charge with $\rho_s = 10^{-8}$ C/m^2 is located at $x = 0$. A positive point charge of 10^{-8} C is at (½, 0, 0) in rectangular coordinates. What is the total force acting on this point charge?

10.5 Charge is uniformly distributed in the spherical volume $r \leq a$ with $\rho_v = 2 \times 10^{-6}$ C/m^3 and $\rho_v = 0$ for $r > a$.

(a) Use Gauss' law to find $\boldsymbol{E}$ for $r \leq a$.
(b) Find the force acting on a test charge of 10^{-12} C at $r = a/2$.
(c) Is the force obtained in (b) to be changed if the charge distribution extends to $r = 2a$ instead of being limited to $r \leq a$?

10.6 In a seed sorting machine, undesirable seeds are deposited with an electrostatic charge while they pass an automatic color-sensitive or size-sensitive monitor. The good seeds are passed uncharged. All seeds are dropped between a high-voltage parallel-plate region to sort out the undesirable seeds. Let the charge on the undesirable seed be q, its mass be m, the voltage between the parallel plates be V, and the plate separation be d. Assume that the seeds enter the parallel-plate region at velocity v_0, and find the displacement y of the bad seed as a function of x. Figure P10.6 illustrates this situation. Consider only the trajectory inside the parallel-plate.

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*P. Lorrain and D. R. Corson, *Electromagnetism* (San Francisco: W. H. Freeman and Co., 1978), p. 189.

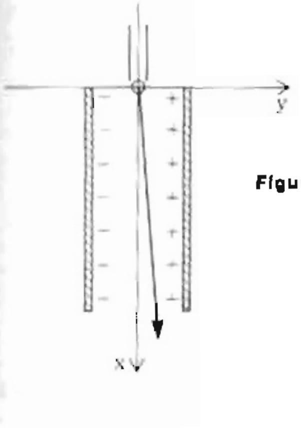


Figure P10.6

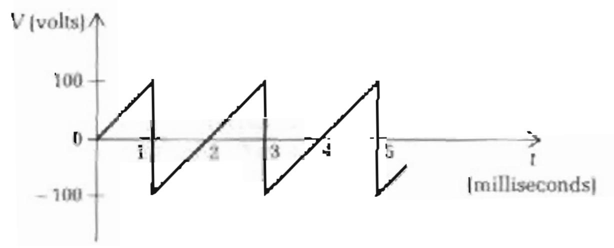


Figure P10.9

10.7 At room temperature (20°C) and standard atmosphere, what should be the size of the corona wire if $b = 3$ cm, $V_0 = 10$ kV, and the roughness factor of the wire is equal to 0.8? (Refer to Figure 10.4.)

10.8 What should the lowest voltage on a Van de Graaff generator be in order to have it produce corona on its surface? Assume that $E_c = 4 \times 10^6$ V/m and that the radius of the metal sphere is equal to 0.6 m.

10.9 Refer to Figure 10.1. If the voltage applied to the parallel plate is the sawtooth signal shown in Figure P10.9, find the locus of the electron on the fluorescent screen located at $x = 20$ cm.

10.10 For the cathode-ray tube shown in Figure 10.8, what should the voltages V_x and V_y be in order to make the electron beam trace a circular path on the screen at 60 revolutions per second? Assume that the vertical and the horizontal deflection plates are identical.

10.11 An electron is accelerated by a difference in potential of 1 kV between the anode and the cathode. It enters the parallel-plate region with this kinetic energy. Its velocity makes a 5° angle with the plane of the parallel plate at the entrance end, as shown in Figure P10.11.

(a) Find v_0, v_{0z}, and v_{0x} at $t = 0$.
(b) Obtain two equations for the coordinates of the electron (x, z) as functions of t. Note that $x = 0$ and $z = 0$ at $t = 0$.
(c) Find the position of the electron at the exit end of the parallel plate.

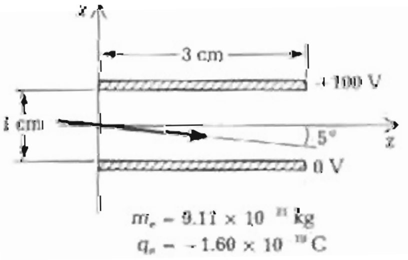

x
3 cm
+100 V
1 cm
5°
z
0 V
$m_e = 9.11 \times 10^{-31}$ kg
$q_e = -1.60 \times 10^{-19}$ C

Figure P10.11

10.12 Consider the ink-jet printer shown in Figure 10.10. Define

$-q_d$ = charge on the ink drop
m_d = mass of the drop
V_0 = deflection-plate voltage
d = deflection-plate spacing
v_d = velocity of the ink drop at entry to the deflection plate
ℓ_{dp} = deflection-plate length
z_p = distance from the deflection-plate entry to the print plane

Show that the vertical displacement of the ink drop is given by

$$x_d = \frac{q_d V_0 \ell_{dp}}{m_d d v_d^2}\left(z_p - \frac{1}{2}\ell_{dp}\right)$$

10.13 Find the capacitance of the spherical capacitor shown in Figure 10.13 by using (10.42) and (10.50). Start from

$$\mathbf{E} = \frac{Q}{4\pi\epsilon r^2}\hat{\mathbf{r}} \quad \text{for} \quad b > r > a$$

and show that your result agrees with (10.47).

10.14 Find the capacitance of the cylindrical capacitor shown in Figure 10.14 by using (10.42) and (10.50). Start from

$$\mathbf{E} = \frac{\rho_\ell}{2\pi\epsilon\rho}\hat{\rho} \quad \text{for} \quad b > \rho > a$$

and show that your result agrees with (10.49).

10.15 Consider the parallel-plate capacitor shown in Figure 10.12. What is the maximum capacitance one can obtain by using mica as the insulator? Let the area of the plate be 10 cm² and the voltage rating of the capacitor be 2 kV, with a safety factor of 10. Use Table 10.1 for the value of ϵ for mica.

10.16 Consider the cylindrical capacitor shown in Figure 10.14. What is the maximum capacitance one can obtain by using oil as the insulator? Take $a = 1$ cm, $h = 2$ cm and the voltage rating = 2 kV, with a safety factor of 5. Use Table 10.1 for the value of ϵ for oil.

10.17 A parallel-plate capacitor is filled with two dielectric materials in a configuration shown in Figure P10.17. The total area of the plate is A. (a) Find the capacitance C in terms of A, d, ϵ_1, and ϵ_2. (b) Suppose that the positive plate carries Q coulombs of charge, and find Q_1 and Q_2 in terms of Q, where Q_1 and Q_2 are charges on the left- and on the right-hand sides of the plate, respectively. Neglect fringing fields.

10.18 Consider the capacitor shown in Figure P10.17. Let $\epsilon_1 = 3\epsilon_0$, $\epsilon_2 = 5\epsilon_0$, $d = 0.6$ mm, and $A = 20$ cm². The potential between the plates is 300 V. Find the total stored electric energy in this capacitor.

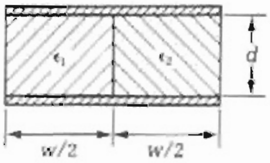


Figure P10.17

3 by using (10.42)

e 10.14 by using

is the maximum
ea of the plate be
factor of 10. Use

is the maximum
cm, $h = 2$ cm and
the value of ϵ for

a configuration
capacitance C in
s Q coulombs of
arges on the left-
ing fields.

$d = 0.6$ mm, and
al stored electric

10.19 Find the capacitance per unit length of a coaxial capacitor with two layers of insulating materials, as shown in Figure 10.15c. Express C/h in terms of a, b, c, ϵ_1, and ϵ_2.

10.20 Find the capacitance C of a parallel-plate capacitor with two layers of insulating materials, as shown in Figure P10.20. Express C in terms of A (the area of the plate), d_1, d_2, ϵ_1, and ϵ_2.

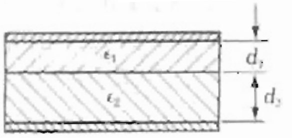


Figure P10.20

10.21 Refer to the capacitor shown in Figure P10.20. Let $\epsilon_1 = 3\epsilon_0$, $\epsilon_2 = 5\epsilon_0$, $d_1 = 0.3$ mm, $d_2 = 0.3$ mm, and $A = 20$ cm^2. The voltage across the capacitor is 300 V. Find the total stored electric energy in this capacitor.

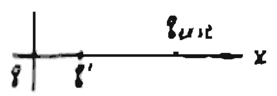
10.22 Derive (10.62).

10.23 A parallel-plate capacitor carries $+Q$ on one plate and $-Q$ on the other plate. The area of each plate is A and the separation between the plates is S. The medium is air.

(a) Find the total stored energy U_E in this capacitor in terms of Q, A, S and ϵ_0.
(b) What is the electrostatic force acting on the plates? Is it attractive or repulsive? **Hint:** find the change in U_E with respect to S.

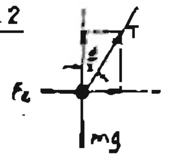
CHAPTER 10

10.1



$$\frac{q}{4\pi\varepsilon(3)^2}+\frac{q'}{4\pi\varepsilon(2)^2}=0 \Rightarrow q'=-\frac{4}{9}q$$

10.2



at equilibrium: $T\sin(\frac{\alpha}{2})=F_e=\frac{q^2}{4\pi\varepsilon(2\ell\sin\frac{\alpha}{2})^2}$ and $T\cos(\frac{\alpha}{2})=mg$

$\therefore \tan(\frac{\alpha}{2})=\frac{q^2}{mg4\pi\varepsilon(2\ell\sin\frac{\alpha}{2})^2}$

Since $\alpha\ll1$, $\tan(\frac{\alpha}{2})\sim\frac{\alpha}{2}$, $\sin(\frac{\alpha}{2})\sim\frac{\alpha}{2}$ and $\frac{\alpha}{2}\sim\frac{q^2}{16\pi\varepsilon\ell^2mg(\alpha/2)^2}$

$\therefore (\frac{\alpha}{2})^3\sim\frac{q^2}{16\pi\varepsilon\ell^2mg}=\frac{(10^{-9})^2\times36\pi\times10^9}{16\pi\times(0.15)^2\times10^{-3}\times9.8}=10.2\times10^{-6}$

$\therefore \frac{\alpha}{2}\sim2.17\times10^{-2}$ rad $\Rightarrow \alpha\sim4.38\times10^{-2}$ rad $=2.49^\circ$

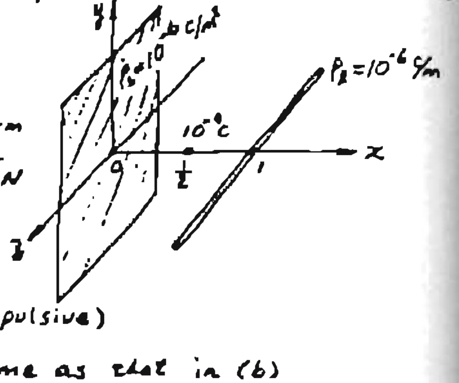
10.3 Gauss' law: $D_\rho=\frac{\rho_\ell}{2\pi\rho}$ $\therefore \bar{E}(\rho)=\frac{\rho_\ell}{2\pi\varepsilon\rho}\hat{\rho}$ and $\bar{F}=q\bar{E}=\frac{(-10^{-9})\times10^{-6}\times36\pi\times10^9}{2\pi\times1}=-1.8\times10^{-5}\hat{\rho}$ N (attractive)

10.4 Force due to plane charge: $\bar{F}_s=q\frac{\rho_s}{2\varepsilon}\hat{x}$

Force due to line charge: $\bar{F}_\ell=q(\frac{\rho_\ell}{2\pi\varepsilon x})(-\hat{x})$

Total force on q: $\bar{F}=\bar{F}_s+\bar{F}_\ell=\hat{x}q(\frac{\rho_s}{2\varepsilon}-\frac{\rho_\ell}{2\pi\varepsilon x})$; $q=10^{-9}$C, $x=0.5$m

$\therefore \bar{F}=\hat{x}\frac{q}{2\varepsilon}(\rho_s-\frac{\rho_\ell}{\pi x})=\hat{x}\frac{10^{-9}}{2}\times36\pi\times10^9(10^{-6}-\frac{10^{-6}}{0.5\pi})=\hat{x}\,2.055\times10^{-5}$N



10.5 (a) For $r\leq a$, $D_r(4\pi r^2)=\rho_v(\frac{4}{3}\pi r^3)\Rightarrow D_r=\frac{\rho_v}{3}r$

$\therefore E_r=\frac{D_r}{\varepsilon}=\frac{\rho_v}{3\varepsilon}r=\frac{2\times10^{-6}}{3}\times36\pi\times10^9\times r=(2.4\pi\times10^4)r$

(b) $\bar{F}(\frac{a}{2})=10^{-12}\bar{E}(\frac{a}{2})=\hat{r}10^{-12}\times2.4\pi\times10^4\times\frac{a}{2}=\hat{r}\,1.2\pi a\times10^{-8}$N (repulsive)

(c) No, since the amount of charges enclosed is the same as that in (b)

10.6 initial conditions: $v_x(0)=v_0$, $v_y(0)=0$; $x(0)=y(0)=0$

$m\frac{dv_x}{dt}=mg\Rightarrow v_x(t)=gt+v_0\Rightarrow x(t)=\frac{g}{2}t^2+v_0t\Rightarrow t^2+\frac{2v_0}{g}t-\frac{2}{g}x=0\Rightarrow t=-\frac{v_0}{g}+\sqrt{\frac{v_0^2}{g^2}+\frac{2x}{g}}$

$m\frac{d^2y}{dt^2}=F_y=q(-\frac{V}{d})\Rightarrow\frac{d^2y}{dt^2}=\frac{-qV}{md}\Rightarrow y(t)=-\frac{qV}{2md}t^2$ $\therefore y(x)=-\frac{qV}{2md}\frac{1}{g^2}\{\sqrt{v_0^2+2gx}-v_0\}^2$

10.7 For $a<r<b$, $E_r(r)=\frac{V}{\ln(b/a)}\frac{1}{r}$ and $E_r^{max}=E_r(r=a)=\frac{V}{\ln(b/a)}\cdot\frac{1}{a}$, where $V=10^4$V; $b=3$cm

To produce corona, $E_r^{max}\geq E_c=3\times10^6m(\delta+3\times10^{-2}\sqrt{\delta/a})$, where $\delta=\frac{298}{293}$ and $m=0.8$

$\therefore \frac{10^4}{\ln(0.03/a)}\frac{1}{a}=3\times10^6\times0.8\times(\frac{298}{293}+3\times10^{-2}\sqrt{\frac{298}{293}\cdot\frac{1}{a}})$

$\Rightarrow 1+855.936(a)+244.096(a\ln a)+25.462(\sqrt{a})+7.261(\sqrt{a}\ln a)=0\Rightarrow a\sim0.36$ mm

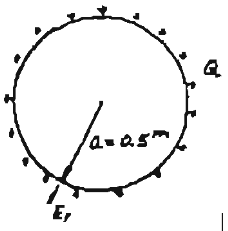
10.8 For $r\geq a$, $E_r=\frac{Q}{4\pi\varepsilon r^2}$

$\therefore V(r\geq a)=-\int_\infty^rE_rdr=-\int_\infty^r\frac{Q}{4\pi\varepsilon r^2}dr=\frac{Q}{4\pi\varepsilon r}$

$\therefore E_r=\frac{V}{r}\Rightarrow E_r^{max}=\frac{V}{a}$

To produce corona on the $r=a$ surface requires

$E_r^{max}=\frac{V}{a}\geq E_c=4\times10^6$ V/m $\Rightarrow V\geq aE_c=0.6\times4\times10^6=2.4\times10^6$ V



10.9 $\tan(-9.9^\circ)=\frac{z^-}{20-4}\Rightarrow z^-=(20-4)\tan(-9.9^\circ)=-2.79$cm

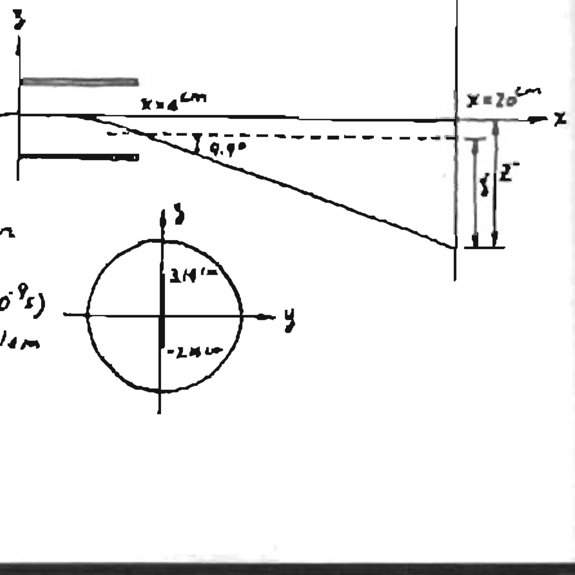
$\therefore Z^-=-2.79-0.35=-3.14$ cm

$\tan(9.9^\circ)=\frac{z^+}{20-4}\Rightarrow z^+=(20-4)\tan(9.9^\circ)=2.79$cm

$\therefore Z^+=279+0.35=3.14$ cm

∴ The locus on the screen is a straight line on the z axis from +3.14 cm to − 3.14 cm

[Since electron takes very short time ($t_0=\frac{0.04}{2\times10^7}=2\times10^{-9}$s) to pass through the parallel plates region, this problem can still be solved by static approach.]



10.10 If $V_x = V_0 \cos\omega t$ and $V_y = V_0 \sin\omega t$, then $x = \frac{q_e V_x}{2m_e d} t_0^2 = \frac{q_e V_0}{2m_e d} t_0^2 \cos\omega t$ and $y = \frac{q_e V_y}{2m_e d} t_0^2 = \frac{q_e V_0}{2m_e d} t_0^2 \sin\omega t$ form a circular path on the screen, where t_0 is the time the electron in the deflection plates region, $d_1 = d_2 = d$ and $\omega = 2\pi \times 60 = 12$

10.11 (a) $\frac{1}{2} m_e v_0^2 = q_e V \Rightarrow v_0 = \sqrt{\frac{2 q_e V}{m_e}} = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 10^3}{9.11\times 10^{-31}}} = 1.874\times 10^7\ m/s$

$v_{0z} = v_z(0) = v_0 \cos 5^\circ = 1.874\times 10^7 \times \cos 5^\circ = 1.867\times 10^7\ m/s$

$v_{0x} = v_x(0) = -v_0 \sin 5^\circ = -1.874\times 10^7 \times \sin 5^\circ = -1.633\times 10^6\ m/s$

(b) $E_x = \frac{V}{d} = -\frac{100}{0.01} = -10^4\ V/m$; $x(0) = z(0) = 0$

$m_e \frac{dv_x(t)}{dt} = q_e E_x \Rightarrow \frac{dv_x(t)}{dt} = \frac{q_e E_x}{m_e} \rightarrow v_x(t) = \frac{q_e E_x}{m_e} t + v_{0x} = \frac{dx(t)}{dt} \Rightarrow x(t) = \frac{q_e E_x}{2m_e} t^2 + v_{0x} t$

$m_e \frac{dv_z(t)}{dt} = 0 \Rightarrow v_z(t) = v_{0z} = \frac{dz(t)}{dt} \longrightarrow z(t) = v_{0z} t$

$\therefore x(t) = \frac{-1.6\times 10^{-19}\times(-10^4)}{2\times 9.11\times 10^{-31}} t^2 - 1.633\times 10^6 t = 8.782\times 10^{14} t^2 - 1.633\times 10^6 t$

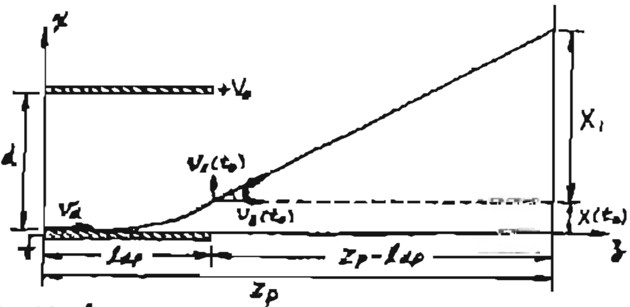
$z(t) = 1.867\times 10^7 t$

(c) $t_0 = l/v_{0z} = 3\times 10^{-2}/1.867\times 10^7 = 1.61\times 10^{-9}\ s$

$\therefore x(t_0) = 8.782\times 10^{14}\times(1.61\times 10^{-9})^2 - 1.633\times 10^6 \times 1.61\times 10^{-9} = -3.53\times 10^{-4}\ m$

$z(t_0) = 1.867\times 10^7 \times 1.61\times 10^{-9} = 3\times 10^{-2}\ m$

10.12 For $0 \le z \le l_{dp}$, $m_d \frac{d^2x}{dt^2} = (-q_d)(-\frac{V_0}{d}) = q_d \frac{V_0}{d}$



$\therefore \frac{d^2x(t)}{dt^2} = \frac{q_d V_0}{m_d d} \Rightarrow x(t) = \frac{q_d}{2m_d} \frac{V_0}{d} t^2$

$t_0 = l_{dp}/v_d$

$\therefore x(t_0) = \frac{q_d}{2m_d} \frac{V_0}{d} (\frac{l_{dp}}{v_d})^2$

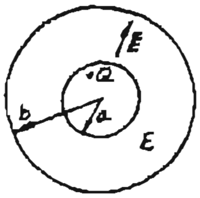
$v_x(t) = \frac{q_d}{m_d} \frac{V_0}{d} t$; $v_z(t) = v_d$

$\therefore v_x(t_0) = \frac{q_d}{m_d} \frac{V_0}{d} (\frac{l_{dp}}{v_d})$; $v_z(t_0) = v_d$

$\tan\theta = \frac{v_x(t_0)}{v_z(t_0)} = \frac{q_d V_0 l_{dp}}{m_d d v_d^2} = \frac{X_1}{z_p - l_{dp}} \Rightarrow X_1 = \frac{q_d}{m_d} \cdot \frac{V_0}{d} \cdot \frac{l_{dp}}{v_d^2} (z_p - l_{dp})$

$\therefore x_d = x(t_0) + X_1 = \frac{q_d}{2m_d} \frac{V_0}{d} (\frac{l_{dp}}{v_d})^2 + \frac{q_d}{m_d} \frac{V_0}{d} \frac{l_{dp}}{v_d^2} (z_p - l_{dp}) = \frac{q_d}{m_d} \frac{V_0}{d} \frac{l_{dp}}{v_d^2} (z_p - l_{dp} + \frac{l_{dp}}{2}) = \frac{q_d V_0 l_{dp}}{m_d d v_d^2} (z_p - \frac{l_{dp}}{2})$

10.13 $E = \frac{Q}{4\pi\epsilon r^2} \hat{r}$ for $a < r < b$



$U_E = \frac{\epsilon}{2} \int_a^b \int_0^\pi \int_0^{2\pi} (\frac{Q}{4\pi\epsilon r^2})^2 r^2 \sin\theta\, dr\, d\theta\, d\phi$

$= \frac{\epsilon}{2} (\frac{Q}{4\pi\epsilon})^2 \times 4\pi \int_a^b \frac{dr}{r^2} = \frac{Q^2}{8\pi\epsilon} (\frac{1}{a} - \frac{1}{b})$

$V_{AB} = -\int_b^a \frac{Q}{4\pi\epsilon r^2} dr = \frac{Q}{4\pi\epsilon} (\frac{1}{a} - \frac{1}{b})$

$U_E = \frac{1}{2} C (V_{AB})^2 \Rightarrow C = \frac{2U_E}{(V_{AB})^2} = \frac{\frac{Q^2}{4\pi\epsilon}(\frac{1}{a} - \frac{1}{b})}{(\frac{Q}{4\pi\epsilon})^2 (\frac{1}{a} - \frac{1}{b})^2} = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$

10.14 $\bar{E} = \frac{\rho_l}{2\pi\epsilon\rho} \hat{\rho}$ for $a < \rho < b$

$U_E = \frac{\epsilon}{2} \int_a^b \int_0^{2\pi} \int_0^h (\frac{\rho_l}{2\pi\epsilon})^2 \frac{1}{\rho^2} \rho\, d\rho\, d\phi\, dz = \frac{\rho_l^2}{4\pi\epsilon} h \ln(b/a)$

$V_{AB} = -\int_b^a \frac{\rho_l}{2\pi\epsilon\rho} d\rho = \frac{\rho_l}{2\pi\epsilon} \ln(b/a)$ and $U_E = \frac{C}{2} (V_{AB})^2 \Rightarrow C = \frac{2U_E}{(V_{AB})^2} = \frac{2\pi\epsilon h}{\ln(b/a)}$

10.15 From Table 10.1, using safety factor 10, $E^{max} = 2\times 10^7$ for mica

$d = V/E = 2\times 10^3 / 2\times 10^7 = 10^{-4}\ m$; $\epsilon(\text{mica}) = 6\epsilon_0$ (Table 10.1)

$\therefore C = \frac{\epsilon A}{d} = \frac{6\times 10\times 10^{-4}}{10^{-4}\times 36\pi\times 10^9} = 5.3\times 10^{-10}\ F$

10.16 From Table 10.1, using safety factor 5, $E^{max} = 3\times10^{6}$ for oil

In the cylinder capacitor, E field everywhere must be smaller than E^{max}.

From (10.25), $E_\rho = \frac{V}{\ln(b/a)}\frac{1}{\rho}$; the strongest E field is at $\rho = a$.

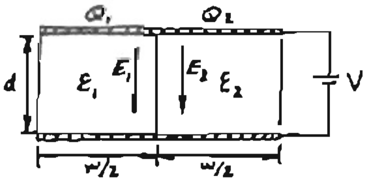
$\therefore$ requires $E^{max} \geq \frac{V}{\ln(b/a)}\frac{1}{a} \Rightarrow \ln(b/a) \geq \frac{V}{aE^{max}} = \frac{2\times10^{3}}{10^{-2}\times3\times10^{6}} = \frac{1}{15}$

From (10.49), $C = 2\pi\varepsilon h/\ln(b/a)$; C is inversely propotional to $\ln(b/a)$.

$\therefore$ To obtain maximum C, the minimum value of $\ln(b/a)$ must be used.

$\therefore C = \frac{2\pi\times2.3\times2\times10^{-2}}{(1/15)\times36\pi\times10^{9}} = 3.83\times10^{-11}F$, where $\varepsilon = 2.3\,\varepsilon_0$ for oil (Table 10.1)

10.17 Assume voltage V across plates, then $E_1 = E_2 = V/d$



(a) $Q_1 = \varepsilon_1E_1\frac{A}{2}$, $Q_2 = \varepsilon_2E_2\frac{A}{2} \rightarrow Q = Q_1+Q_2 = (\varepsilon_1+\varepsilon_2)\frac{V}{d}\frac{A}{2}$

$\therefore C = Q/V = \frac{A}{2d}(\varepsilon_1+\varepsilon_2)$

(b) $Q_1/Q_2 = \varepsilon_1/\varepsilon_2$ and $Q = Q_1+Q_2 \Rightarrow Q_1 = \frac{\varepsilon_1Q}{\varepsilon_1+\varepsilon_2}$ and $Q_2 = \frac{\varepsilon_2Q}{\varepsilon_1+\varepsilon_2}$

10.18 $U_E = \frac{1}{2}CV^2 = \frac{A}{4d}(\varepsilon_1+\varepsilon_2)V^2 = \frac{20\times10^{-4}}{4\times6\times10^{-4}}\times\frac{1}{36\pi}\times10^{-9}\times(3+5)\times(300)^2 = 5.3\times10^{-6}$ Joule

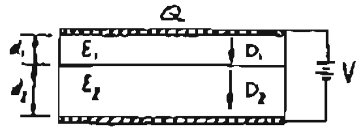
10.19 From (10.22), $E_\rho = \frac{\rho_\ell}{2\pi\varepsilon_1\rho}$ for $a<\rho<c$; $E_\rho = \frac{\rho_\ell}{2\pi\varepsilon_2\rho}$ for $c<\rho<b$

$V = V_{ac}+V_{cb} = -\int_c^a \frac{\rho_\ell}{2\pi\varepsilon_1\rho}d\rho - \int_b^c \frac{\rho_\ell}{2\pi\varepsilon_2\rho}d\rho = \frac{\rho_\ell}{2\pi\varepsilon_1}\ln(c/a) + \frac{\rho_\ell}{2\pi\varepsilon_2}\ln(b/c)$

$\therefore C = \frac{Q}{V} = (\rho_\ell h)/\left\{\frac{\rho_\ell}{2\pi\varepsilon_1}\ln(c/a)+\frac{\rho_\ell}{2\pi\varepsilon_2}\ln(b/c)\right\} = \frac{2\pi h}{\frac{1}{\varepsilon_1}\ln(c/a)+\frac{1}{\varepsilon_2}\ln(b/c)}$

$\therefore C/h = 2\pi/\left\{\frac{1}{\varepsilon_1}\ln(c/a)+\frac{1}{\varepsilon_2}\ln(b/c)\right\}$

10.20 Assume voltage V across plates, then $D_1 = D_2$



$\therefore \varepsilon_1E_1 = \varepsilon_2E_2 \Rightarrow E_1 = \frac{\varepsilon_2}{\varepsilon_1}E_2$

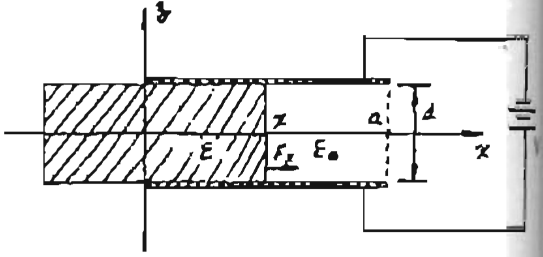
$Q = \varepsilon_1E_1A = \varepsilon_2E_2A$

$V = E_1d_1+E_2d_2 = E_2\left(\frac{\varepsilon_2}{\varepsilon_1}d_1+d_2\right) = E_2\left(\frac{\varepsilon_2d_1+\varepsilon_1d_2}{\varepsilon_1}\right)$

$\therefore C = \frac{Q}{V} = \frac{\varepsilon_1\varepsilon_2A}{(\varepsilon_2d_1+\varepsilon_1d_2)} = \frac{A}{(d_1/\varepsilon_1+d_2/\varepsilon_2)}$

10.21 $U_E = \frac{1}{2}CV^2 = \frac{20\times10^{-4}\times(300)^2}{2\times(3\times10^{-4}/3+3\times10^{-4}/5)\times36\pi\times10^{9}} = 4.97\times10^{-6}$ Joule

10.22 $E = V/d$ and E (in air) $= E$ (in slab)



Slab at x, $Q_x = \varepsilon Exh+\varepsilon_0E(a-x)h$

slab at $x+\Delta x$, $Q_{x+\Delta x} = \varepsilon E(x+\Delta x)h+\varepsilon_0E(a-x-\Delta x)h$

$\therefore \Delta Q = Q_{x+\Delta x}-Q_x = \varepsilon E\Delta xh-\varepsilon_0E\Delta xh = Eh\Delta x(\varepsilon-\varepsilon_0)$

$\therefore$ Battery spends store energy by the amount of

$\Delta U_B = \Delta Q\cdot V = Eh\Delta x(\varepsilon-\varepsilon_0)V = \frac{V^2}{d}h\Delta x(\varepsilon-\varepsilon_0)$

Store energy increased due to the increasing of Δx is

$\Delta U_E = \frac{\varepsilon}{2}\frac{V}{d}\cdot V\Delta xh-\frac{\varepsilon_0}{2}\frac{V}{d}V\Delta xh = \frac{1}{2}\frac{V^2}{d}h\Delta x(\varepsilon-\varepsilon_0)$

$\therefore \Delta U = \Delta U_E-\Delta U_B = -\frac{1}{2}\frac{V^2}{d}h\Delta x(\varepsilon-\varepsilon_0)$

$\therefore F_x = -\frac{\Delta U}{\Delta x} = \frac{V^2}{2d}h(\varepsilon-\varepsilon_0)$

10.23 (a) $\rho_s = \dfrac{Q}{A}$

$$D = \rho_s = \frac{Q}{A}$$

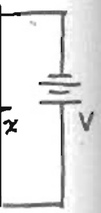
$$E = \frac{D}{\varepsilon} = \frac{Q}{A\varepsilon_0}$$

$$U_E = \frac{1}{2}\varepsilon_0\left(\frac{Q^2}{A^2\varepsilon_0^2}\right)(As)$$

$$= \frac{Q^2 s}{2A\varepsilon_0}$$

(b) $F = -\dfrac{dU_E}{ds} = \dfrac{-Q^2}{2A\varepsilon_0}$

force tends to decrease U_E. Therefore the force is attractive.



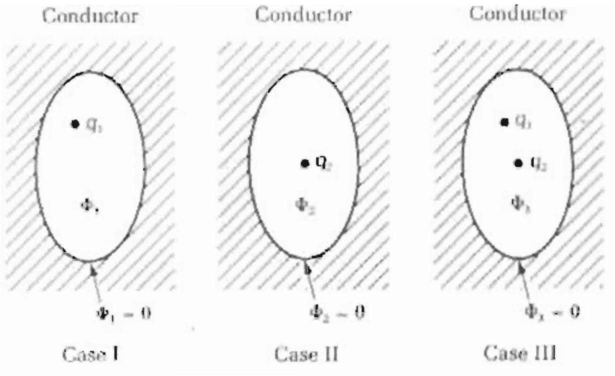


Figure P11.1

Problems

11.1 Consider the three boundary-value problems shown in Figure P11.1. The solution of Case I is Φ_1, and the solution of Case II is Φ_2. In Case III, the charges q_1 and q_2 are the same charges that appear in cases I and II, and they appear in exactly corresponding positions. Express Φ_3 in terms of Φ_1 and Φ_2.

11.2 Consider the three boundary-value problems shown in Figure P11.2. The solution of Case I is Φ_1, and the solution of Case II is Φ_2. In Case III, the charges q_1 and q_2 are the same charges that appear in cases I and II, and they appear in exactly corresponding positions. Note the differences in the boundary conditions for the three cases. Can Φ_3 be expressed in terms of Φ_1 and Φ_2? If so, obtain the expression. If not, explain why.

11.3 The radius of the inner conductor of a coaxial line is a and that of the outer conductor is b. The potential of the inner conductor is V and that of the outer conductor is zero. There is no volume charge density between b and a. Start from the Laplace equation to obtain the potential in the coaxial line.

11.4 Two concentric conducting spheres have radii a and b, respectively ($b > a$). The outer sphere is at zero potential, and the inner sphere is maintained at V volts. There is no space charge between the conductors. Start from the Laplace equation to obtain the potential $\Phi(r)$ for $b > r > a$.

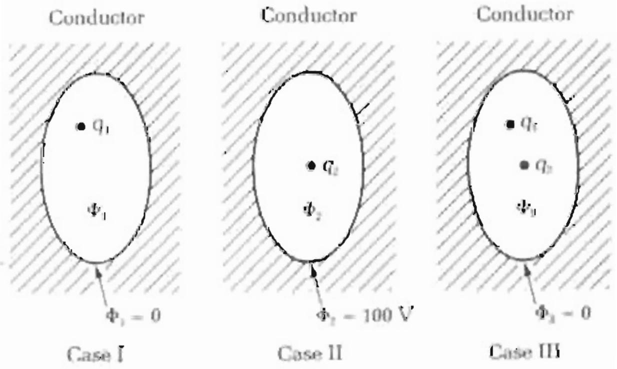
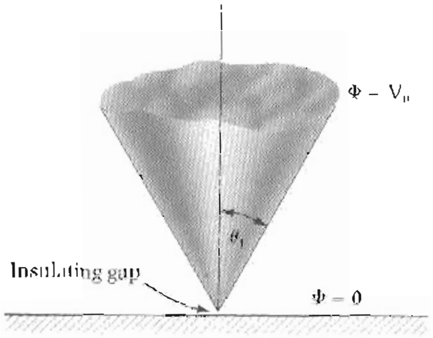


Figure P11.2



11.5 In Figure P11.5 a conducting cone is at a potential V_0, and a small gap separates its vertex from a conducting plane. The axis of the cone is perpendicular to the conducting plane, which is maintained at zero potential. The angle of the cone is θ_1. Because of the symmetry of this problem and the fact that the boundary conditions on the potential Φ involve θ only, Φ is independent of r and ϕ when spherical coordinates are used. Find the potential $\Phi(\theta)$ in the region $\theta_1 \leq \theta \leq 90°$. **Hint:** $\int (1/\sin\theta)\,d\theta = \ln(\tan\theta/2)$. Find the surface charge density on the cone.

11.6 The upper plate of the parallel-plate capacitor discussed in Example 11.1 is maintained at 100 V, and the lower plate is at 80 V. All other conditions remain unchanged. Find Φ.

11.7 Model a dc vacuum-tube rectifier as two parallel plates with a space charge in between, as shown in Figure 11.3. Let the separation be 1 cm. Find the voltage needed to produce 1 A/m^2 current.

11.8 Find the surface charge distribution on the vertical and the horizontal conducting walls for the case discussed in Example 11.8. Plot ρ_s for $z \to 0$ and $x = y = 0$. Let $q = 10^{-8}$ C, and $a = b = 1$.

11.9 Find the images of a point charge near a corner of a conductor similar to the one shown in Figure 11.11 except that $\phi_0 = 45°$.

11.10 Find the electrostatic force that acts on the point charge q at $(0, 0, d)$ and is due to induced surface charges at $z = 0$, as shown in Figure 11.5.

11.11 Calculate the capacitance per meter of a 12-inch (0.3048 m)-diameter steel pipe located 6 ft (1.83 m) above and parallel to the ground.

11.12 Example 11.10 states that the maximum electric field on the surface of the conducting cylinder is located at the point nearest the ground. Show the validity of this statement by plotting out E_r on the surface as a function of ϕ. Use the following data: $V_0 = 100$ V, $b = 2$ m, and $a = 1$ m.

11.13 For the point charge q located d meters from a grounded conducting sphere shown in Figure 11.14, find the surface charge distribution as a function of θ.

11.14 Repeat the preceding problem for an isolated conducting sphere carrying no net charge.

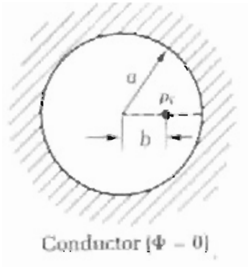


Figure P11.16

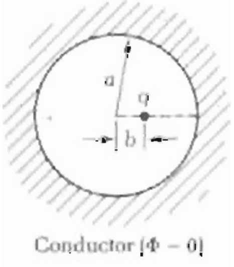


Figure P11.18

11.15 Equation (11.42) gives the potential due to a point charge in the presence of a **grounded** conducting sphere. Equation (11.44) gives the potential due to a point charge in the presence of an **isolated** sphere carrying **no** net charge. From these results, find the potential due to a point charge q, d meters from an isolated conducting sphere carrying a net charge of q_0.

11.16 A line charge ρ_ℓ is inside a conducting tunnel of radius a, as shown in Figure P11.16. Notice that the line charge is b meters off center. Find the potential function in the tunnel. **Hint:** This is a complementary problem of the one shown in Figure 11.12.

11.17 Calculate the force per meter acting on the line charge in the tunnel shown in Figure P11.16.

11.18 A point charge q is inside a spherical cavity of a conductor, as shown in Figure P11.18. The radius of the cavity is a and the cavity is filled with air.

(a) Find the potential Φ in the cavity when $b = 0$.
(b) Find the surface charge density of the cavity wall when $b = 0$.
(c) Find the potential Φ in the cavity when $b = a/2$.
(d) Find the surface charge density of the cavity wall when $b = a/2$.

11.19 Calculate the electrostatic force acting on the point charge in the cavity shown in Figure P11.18.

11.20 Sketch the E lines due to a point charge near the interface of two dielectric media. The situation is similar to the one shown in Figure 11.17, except that $\epsilon_2 = 0.5\epsilon_1$.

11.21 A rectangular conducting trough of width a and height b is maintained at zero potential, as shown in Figure P11.21. The potential on the top plate, which covers the trough, is known to be $\Phi(x, b) = 200 \sin(2\pi x/a)$ volts. Find the potential Φ in the trough. There is no volume charge in the trough.

11.22 Three sides of a rectangular conducting pipe are grounded, while the fourth side is maintained at 100 V, as shown in Figure P11.22. Find the potential in the pipe. There is no volume charge in the pipe.

Figure P11.21

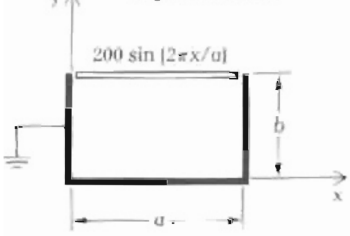
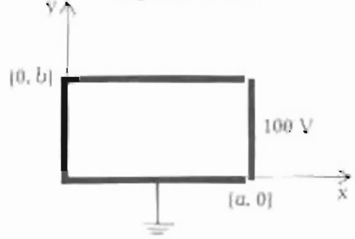


Figure P11.22



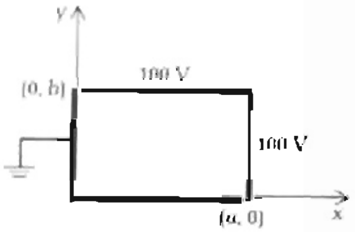


Figure P11.23

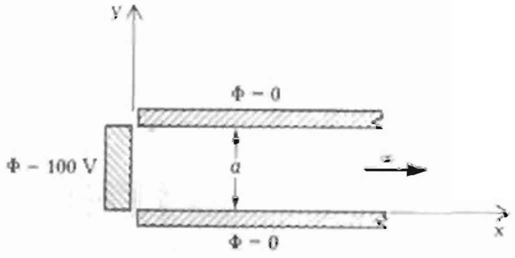


Figure P11.24

11.23 The boundary potentials of a rectangular conducting pipe are shown in Figure P11.23. Find the potential in the pipe. There is no volume charge in the pipe.

11.24 Consider the boundary value problem shown in Figure P11.24. The upper and the lower conducting plates are maintained at zero potential. The plate at the left is maintained at 100 V. Two gaps insulate the side plate from the ground. There is no volume charge in the region and Φ approaches zero as x approaches infinity.

(a) Use the method of separation of variables to obtain two ordinary differential equations.

(b) Solve the differential equations. (The function involving y must be a sine function.)

(c) Match the boundary conditions, and find the final solution.

11.25 A spherical capacitor is filled with a dielectric material of ϵ_1 in half of the space and with another material of ϵ_2 in the remaining space, as shown in Figure P11.25.

(a) Find the potential function $\Phi(r)$ in the region $a<r<b$. The potential at $r = a$ is V_0 and it is zero at $r = b$. **Hint:** The potential satisfies the Laplace equation (11.2) and it may be assumed that it is a function of r only.

(b) Find the electric field in the region $a < r < b$.

(c) Find the $\mathbf{D}$ field in the region $a<r<b$. **Hint:** The $\mathbf{D}$ field in medium 1 is different from that in medium 2. Note: the boundary conditions on the tangential $\mathbf{E}$ and on the normal $\mathbf{D}$ fields are satisfied using the suggested approach.

(d) Find the total charge on the inner conductor and the capacitance of this capacitor.

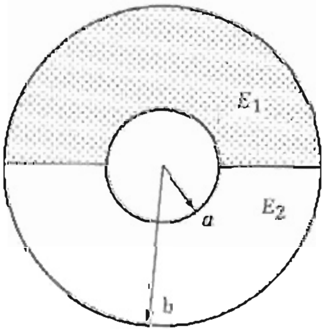


Figure P11.26

CHAPTER 11

11.1 Case I: $\nabla^2\phi_1 = \rho_1/\varepsilon$ and $\phi_1 = 0$ on the boundary

Case II: $\nabla^2\phi_2 = \rho_2/\varepsilon$ and $\phi_2 = 0$ on the boundary

$\therefore \nabla^2(\phi_1+\phi_2) = (\rho_1+\rho_2)/\varepsilon$ with $\phi_1+\phi_2 = 0$ on the boundary $\Rightarrow \phi_3 = \phi_1+\phi_2$ in Case III

11.2 Case I: $\nabla^2\phi_1 = \rho_1/\varepsilon$ and $\phi_1 = 0$ on the boundary

Case II: $\nabla^2\phi_2 = \rho_2/\varepsilon$ and $\phi_2 = 100$ on the boundary

$\Rightarrow \nabla^2(\phi_2 - 100) = \rho_2/\varepsilon$ and $\phi_2 - 100 = 0$ on the boundary

$\therefore \nabla^2(\phi_1+\phi_2-100) = (\rho_1+\rho_2)/\varepsilon$ with $\phi_1+\phi_2-100 = 0$ on the boundary $\Rightarrow \phi_3 = \phi_1+\phi_2-100$ in Cas

11.3 potential ϕ is function of ρ only $\Rightarrow \nabla^2\phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial\phi}{\partial\rho}) = 0 \Rightarrow \phi = C_1 \ln\rho + C_2$

$\phi(a) = V = C_1\ln a + C_2$ and $\phi(b) = 0 = C_1\ln b + C_2 \Rightarrow C_1 = V/\ln(\frac{a}{b})$ and $C_2 = -V\ln b/\ln(\frac{a}{b}$

$\therefore \phi(\rho) = V\ln\rho/\ln(\frac{a}{b}) - V\ln b/\ln(\frac{a}{b}) = V\ln(\frac{\rho}{b})/\ln(\frac{a}{b})$

11.4 $\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\Phi}{\partial r}) = 0 \Rightarrow \Phi(r) = C_1 - C_2/r$

$\Phi(a) = V = C_1 - C_2/a$ and $\Phi(b) = 0 = C_1 - C_2/b \Rightarrow C_1 = \frac{V}{b(1/b - 1/a)}$ and $C_2 = \frac{V}{1/b - 1/a}$

$\therefore \Phi(r) = \frac{V}{1/b - 1/a}(1/b - 1/r)$

11.5 $\nabla^2\Phi(\theta) = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\Phi}{\partial\theta}) = 0 \Rightarrow \frac{\partial\Phi}{\partial\theta} = \frac{C_1}{\sin\theta} \Rightarrow \Phi(\theta) = C_1\ln(\tan\frac{\theta}{2}) + C_2$

$\Phi(\theta_1) = V_0 = C_1\ln(\tan\frac{\theta_1}{2}) + C_2$ and $\Phi(90^\circ) = 0 = C_2 \Rightarrow C_1 = V_0/\ln(\tan\frac{\theta_1}{2})$ and $C_2 = 0$

$\therefore \Phi(\theta) = V_0\ln(\tan\frac{\theta}{2})/\ln(\tan\frac{\theta_1}{2})$ and $\bar{E} = -\nabla\Phi(\theta) = -\hat{\theta}\frac{1}{r}\frac{\partial\Phi}{\partial\theta} = -\hat{\theta}\, V_0/r\ln(\tan\frac{\theta_1}{2})\sin\theta$

$\therefore \rho_s = \varepsilon\bar{E}\cdot\hat{\theta} = -\varepsilon V_0/r\ln(\tan\frac{\theta_1}{2})\sin\theta_1$ (since $\tan\frac{\theta_1}{2} < 1$, $\rho_s > 0$ if $V_0 > 0$)

11.6 $\nabla^2\Phi = \partial^2\Phi/\partial y^2 = 0 \Rightarrow \Phi(y) = C_1 y + C_2$ $\quad \Phi(0) = 80 = C_2$ and $\Phi(0.1) = 0.1C_1 + 80 = 100 \Rightarrow C_1 =$

$\therefore \Phi(y) = 200y + 80$

11.7 $J = \frac{4\varepsilon}{9d^2}\sqrt{2e/m}\, V^{3/2} \Rightarrow 1 = \frac{4\times\frac{1}{36\pi}\times10^{-9}}{9\times(10^{-2})^2}\sqrt{\frac{2\times1.6\times10^{-19}}{9.11\times10^{-31}}}\, V^{3/2} \Rightarrow V = 12.26\ \text{V}$

11.8 $\Phi = \frac{q}{4\pi\varepsilon}[1/R_1 - 1/R_2 + 1/R_3 - 1/R_4]$, where $R_1 = \sqrt{(x-1)^2+y^2+(z-1)^2}$, $R_2 = \sqrt{(x+1)^2+y^2+(z-1)^2}$,

$R_3 = \sqrt{(x+1)^2+y^2+(z+1)^2}$ and $R_4 = \sqrt{(x-1)^2+y^2+(z+1)^2}$

$$\bar{E} = -\nabla\Phi = \frac{q}{4\pi\varepsilon}\left\{\frac{\hat{x}(x-1)+\hat{y}y+\hat{z}(z-1)}{R_1^3} - \frac{\hat{x}(x+1)+\hat{y}y+\hat{z}(z-1)}{R_2^3} + \frac{\hat{x}(x+1)+\hat{y}y+\hat{z}(z+1)}{R_3^3} - \frac{\hat{x}(x-1)+\hat{y}y+\hat{z}(z+1)}{R_4^3}\right\}$$

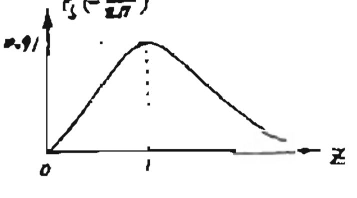
on the Horizontal wall, $z = 0$ and $\bar{E} = \frac{q}{2\pi\varepsilon}\{[(x+1)^2+y^2+1]^{-3/2} - [(x-1)^2+y^2+1]^{-3/2}\}\hat{z}$

$\therefore \rho_s = \varepsilon\bar{E}\cdot\hat{z} = \frac{q}{2\pi}\{[(x+1)^2+y^2+1]^{-3/2} - [(x-1)^2+y^2+1]^{-3/2}\}$

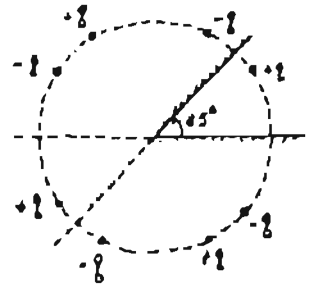
on the vertical wall, $x = 0$ and $\bar{E} = \frac{q}{2\pi\varepsilon}\{[1+y^2+(z+1)^2]^{-3/2} - [1+y^2+(z-1)^2]^{-3/2}\}\hat{x}$

$\therefore \rho_s = \varepsilon\bar{E}\cdot\hat{x} = \frac{q}{2\pi}\{[1+y^2+(z+1)^2]^{-3/2} - [1+y^2+(z-1)^2]^{-3/2}\}$

For $z > 0$, $x = y = 0$ and $q = 10^{-6}$ C, $\rho_s = \frac{10^{-6}}{2\pi}\{[1+(z+1)^2]^{-3/2} - [1+(z-1)^2]^{-3/2}\}$



11.9



11.10 $\bar{F} = \hat{z}\frac{-qq}{4\pi\varepsilon_0(2d)^2} = \hat{z}\frac{-q^2}{16\pi\varepsilon_0 d^2}$

11.11 From (11.40), $C = 2\pi\varepsilon_0/\cosh^{-1}(\frac{h}{a}) = 2\pi\times\frac{1}{36\pi}\times10^{-9}/\cosh^{-1}(\frac{72''}{6''}) = 17.5\ \mu\mu f/m$

11.12 On the surface $E_\phi = 0$. From Example 11.10,

$$\bar{E}\text{ (on the surface)} = \hat{\rho}E_\rho = \hat{\rho}\frac{-\rho_\ell}{2\pi\varepsilon}\left[\frac{\rho-\rho_0\cos\phi}{\rho^2+\rho_0^2-2\rho\rho_0\cos\phi} - \frac{\rho-d\cos\phi}{\rho^2+d^2-2\rho d\cos\phi}\right]$$

$$\therefore E_\rho(a,\phi) = \frac{-\rho_\ell}{2\pi\varepsilon}\left[\frac{a-\rho_0\cos\phi}{a^2+\rho_0^2-2a\rho_0\cos\phi} - \frac{a-d\cos\phi}{a^2+d^2-2ad\cos\phi}\right]$$

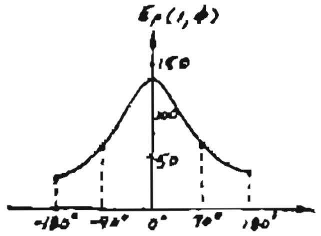
$$= \frac{-\rho_\ell}{2\pi\varepsilon}\left[\frac{1}{2a}\cdot\frac{a/\rho_0-\cos\phi}{(a^2+\rho_0^2)/2a\rho_0-\cos\phi} - \frac{1}{2a}\cdot\frac{a/d-\cos\phi}{(a^2+d^2)/2ad-\cos\phi}\right]$$

$$= \frac{-\rho_\ell}{4\pi\varepsilon a}\left[\frac{d/a - a/d}{(a^2+d^2)/2ad-\cos\phi}\right], \quad \text{since } \rho_0 = a^2/d$$

$\Rightarrow$ max. of E_ρ occurs at $\phi = 0$

$$V_0 = 100^{\text{v}} = \frac{\rho_\ell}{2\pi\varepsilon}\ln(a/d), \quad d = h+\sqrt{h^2-a^2} = 2+\sqrt{4-1} = 3.73$$

$$\therefore E_\rho(a=1,\phi) = \frac{V_0}{2a\ln(d/a)}\left[\frac{d/a-a/d}{(a^2+d^2)/2ad-\cos\phi}\right] = \frac{181.5}{2-\cos\phi}$$



11.13 $$\phi(r,\theta) = \frac{1}{4\pi\varepsilon}\left[\frac{q}{(r^2+d^2-2rd\cos\theta)^{1/2}} + \frac{q'}{(r^2+r_0^2-2rr_0\cos\theta)^{1/2}}\right],$$ where $r_0 = a^2/d$ and $q' = -q(a/d)$

$$E_r = \hat{r}\cdot(-\nabla\phi) = \frac{1}{4\pi\varepsilon}\left[\frac{q(r-d\cos\theta)}{(r^2+d^2-2rd\cos\theta)^{3/2}} + \frac{q'(r-r_0\cos\theta)}{(r^2+r_0^2-2rr_0\cos\theta)^{3/2}}\right]$$

$$E_r\Big|_{r=a} = \frac{1}{4\pi\varepsilon}\left[\frac{qd(a/d-\cos\theta)}{(2ad)^{3/2}\left(\frac{a^2+d^2}{2ad}-\cos\theta\right)^{3/2}} + \frac{q'r_0(a/r_0-\cos\theta)}{(2ar_0)^{3/2}\left(\frac{a^2+r_0^2}{2ar_0}-\cos\theta\right)^{3/2}}\right],$$ where $r_0 = a^2/d$ and $q' = -q(a/d)$

$$= \frac{q}{4\pi\varepsilon}\left[\frac{d}{(2ad)^{3/2}}\cdot\frac{a/d-\cos\theta}{\left(\frac{a^2+d^2}{2ad}-\cos\theta\right)^{3/2}} - \frac{1}{(2a)^{3/2}(d)^{3/2}}\cdot\frac{d/a-\cos\theta}{\left(\frac{a^2+d^2}{2ad}-\cos\theta\right)^{3/2}}\right] = \frac{q}{4\pi\varepsilon}\cdot\frac{d}{(2ad)^{3/2}}\cdot\frac{a/d-d/a}{\left(\frac{a^2+d^2}{2ad}-\cos\theta\right)^{3/2}}$$

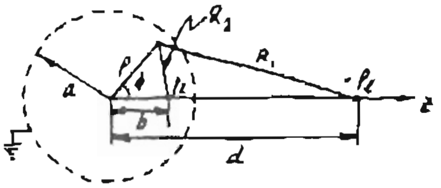
$$= \frac{q}{4\pi\varepsilon a}\cdot\frac{a^2-d^2}{(a^2+d^2-2ad\cos\theta)^{3/2}} \qquad \therefore \rho_s = \varepsilon E_r = \frac{q}{4\pi a}\cdot\frac{a^2-d^2}{(a^2+d^2-2ad\cos\theta)^{3/2}}$$

11.14 $$\rho_s = \frac{q}{4\pi a}\cdot\frac{a^2-d^2}{(a^2+d^2-2ad\cos\theta)^{3/2}} + \frac{(-q')}{4\pi a^2} = \frac{q}{4\pi a}\left[\frac{a^2-d^2}{(a^2+d^2-2ad\cos\theta)^{3/2}} + \frac{1}{d}\right], \quad q' = -q(a/d)$$

11.15 $$\phi = \frac{q}{4\pi\varepsilon}\left[\frac{1}{R_1} - \frac{a}{R_2 d}\right] + \frac{(-q')+q_0}{4\pi\varepsilon r} = \frac{q}{4\pi\varepsilon}\left[\frac{1}{R_1} - \frac{a}{R_2 d} + \frac{a}{rd}\right] + \frac{q_0}{4\pi\varepsilon r},$$ where $R_1 = (r^2+d^2-2rd\cos\theta)^{1/2}$

$R_2 = (r^2+r_0^2-2rr_0\cos\theta)^{1/2}$

11.16



$$R_1 = (\rho^2+d^2-2\rho d\cos\phi)^{1/2}$$

$$R_2 = (\rho^2+b^2-2\rho b\cos\phi)^{1/2}$$

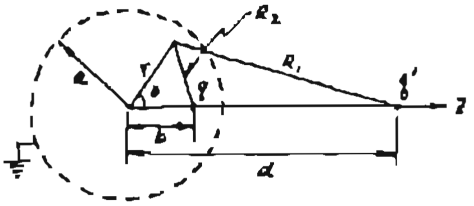
$$\therefore \Phi(\rho,\phi) = -\frac{\rho_\ell}{2\pi\varepsilon}\left[\ln(R_2/R_1) - \ln(a/d)\right] \quad \text{with } d = a^2/b$$

11.17 Apply Gauss' law, $E_\rho = \dfrac{\rho_\ell}{2\pi\varepsilon(d-b)}$ at $\rho = b, \phi = 0$ due to $-\rho_\ell$ with $d = a^2/b$

$$\therefore \bar{F} = \frac{\rho_\ell\cdot\rho_\ell}{2\pi\varepsilon(d-b)}\hat{z} = \frac{\rho_\ell^2}{2\pi\varepsilon}\cdot\frac{b}{a^2-b^2}\hat{z}$$

11.18 (a) $$\Phi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a}\right)$$

(b) $$\rho_s = \frac{q}{4\pi a^2}$$



(c) $$r_0 = b = \frac{a^2}{d} \rightarrow d = \frac{a^2}{b} = 2a$$

$$q' = -q\,\frac{d}{a} = -2q$$

$$\Phi = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{r^2+(\frac{a}{2})^2-ra\cos\theta}} - \frac{2}{\sqrt{r^2+4a^2-4ar\cos\theta}}\right]$$

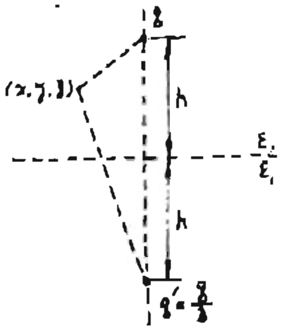
(d) $$E_r = -\frac{\partial\Phi}{\partial r} = \frac{-q}{4\pi\varepsilon_0}\left[\frac{-r+\frac{1}{2}a\cos\theta}{[r^2+(\frac{a}{2})^2-ra\cos\theta]^{3/2}} - \frac{2r-4a\cos\theta}{[r^2+4a^2-4ar\cos\theta]^{3/2}}\right]$$

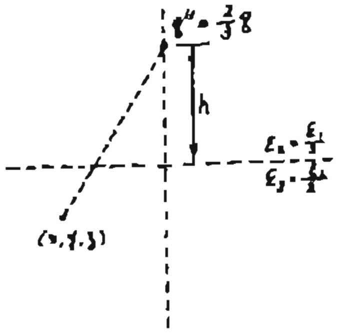
$$\rho_s = -\varepsilon_0 E_r\Big|_{r=a} = \frac{q}{4\pi}\left[\frac{-a+\frac{1}{2}a\cos\theta}{(\frac{5}{4}a^2-a^2\cos\theta)^{3/2}} + \frac{2a-4a\cos\theta}{[5a^2-4a^2\cos\theta]^{3/2}}\right]$$

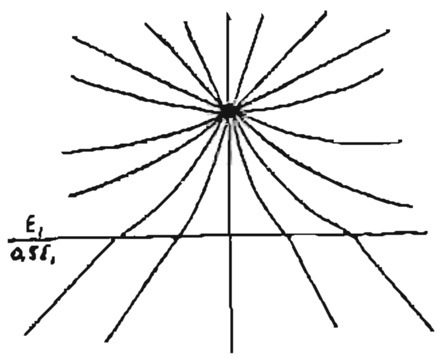
$$= \frac{-3q}{2\pi a^2(5-4\cos\theta)^{3/2}}.$$

11.19 $\bar{F} = \frac{qq'}{4\pi\varepsilon(d-b)^2}\hat{z} = \frac{-abq^2}{4\pi\varepsilon(a^2-b^2)^2}\hat{z}$

11.20







11.21 $\nabla^2\Phi = 0$ and $\frac{\partial}{\partial z} = 0 \Rightarrow \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0 \Rightarrow \Phi(x,y) = (A\cos kx + B\sin kx)(C\cosh ky + D\sinh ky)$

$\Phi(0,y) = 0 \Rightarrow A = 0$, $\Phi(x,0) = 0 \Rightarrow C = 0$ $\therefore \Phi(x,y) = E\sin kx \sinh ky$

$\Phi(a,y) = 0 \Rightarrow k = \frac{m\pi}{a}$, $m = 1,2,3,\cdots$ $\therefore \Phi(x,y) = \sum_{m=1}^{\infty} A_m \sin(\frac{m\pi}{a}x)\sinh(\frac{m\pi}{a}y)$

$\Phi(x,b) = 200\sin(2\pi x/a) = \sum_{m=1}^{\infty} A_m \sin(\frac{m\pi}{a}x)\sinh(\frac{m\pi}{a}b) \Rightarrow m = 2$ and $A_2 = 200/\sinh(\frac{2\pi}{a}b)$

$\therefore \Phi(x,y) = \frac{200}{\sinh(\frac{2\pi}{a}b)}\sin(\frac{2\pi}{a}x)\sinh(\frac{2\pi}{a}y)$

11.22 $\nabla^2\Phi = 0$ and $\frac{\partial}{\partial z} = 0 \Rightarrow \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0 \Rightarrow \Phi(x,y) = (A\cosh kx + B\sinh kx)(C\cos ky + D\sin ky)$

$\Phi(0,y) = 0 \Rightarrow A = 0$, $\Phi(x,0) = 0 \Rightarrow C = 0$ $\therefore \Phi(x,y) \sim \sinh kx \sin ky$

$\Phi(x,b) = 0 \Rightarrow k = \frac{m\pi}{b}$, $m = 1, 2, 3, \cdots$ $\therefore \Phi(x,y) = \sum_{m=1}^{\infty} A_m \sinh(\frac{m\pi}{b}x)\sin(\frac{m\pi}{b}y)$

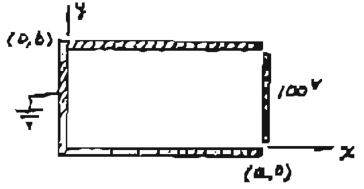
$\Phi(a,y) = 100 = \sum_{m=1}^{\infty} A_m \sinh(\frac{m\pi}{b}a)\sin(\frac{m\pi}{b}y)$, $0 < y < b$

orthogonality: $\int_0^b 100\sin(\frac{m\pi}{b}y)dy = A_m\sinh(\frac{m\pi}{b}a)\int_0^b \sin^2(\frac{m\pi}{b}y)dy = A_m\frac{b}{2}\sinh(\frac{m\pi}{b}a)$

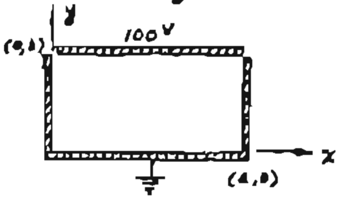
$\int_0^b 100\sin(\frac{m\pi}{b}y)dy = \frac{100b}{m\pi}(1-\cos m\pi) = \begin{cases} 0, & m = 2,4,6,\cdots \\ 200b/m\pi, & m = 1,3,5,\cdots \end{cases}$

$\therefore A_m = \frac{400}{m\pi\sin(m\pi a/b)}$, $m = 1,3,5,\cdots$ and $\Phi(x,y) = \sum_{m=\text{odd}}^{\infty} \frac{400}{m\pi\sinh(m\pi a/b)}\sinh(m\pi x/b)\sin(m\pi y/b)$

11.23 Figure P11.23 is the superposition of the following two cases:



Case 1



Case 2

Case 1 has been solved in Prob. 11.22. Case 2 can be solved by changing the coordinat system of case 1, that is, $\Phi_2(x,y) = \sum_{m=\text{odd}}^{\infty} \frac{400}{m\pi\sinh(m\pi b/a)}\sinh(m\pi y/a)\sin(m\pi x/a)$

Potential $\Phi(x,y)$ in the pipe shown in Figure P11.23 is the combination of case 1 and case that is,

$$\Phi(x,y) = \sum_{m=\text{odd}}^{\infty} \frac{400}{m\pi}\left\{\frac{\sinh(m\pi x/b)\sin(m\pi y/b)}{\sinh(m\pi a/b)} + \frac{\sinh(m\pi y/a)\sin(m\pi x/a)}{\sinh(m\pi b/a)}\right\}$$

11.24 (a) $\nabla^2\Phi = 0$ and $\frac{\partial}{\partial z} = 0 \Rightarrow \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0$ Let $\Phi(x,y) = XY$, then $\frac{X''}{X} + \frac{Y''}{Y} = 0$

$\therefore \frac{X''}{X} = k^2$, $\frac{Y''}{Y} = -k^2$

(b) $X(x) = C_1 e^{-kx} + C_2 e^{kx}$ and $Y(y) \sim \sin ky$

$X(x)$ must be finite as $x \to \infty \Rightarrow C_2 = 0$

(c) $\Phi(x,y) = X(x)Y(y) = Ae^{-kx}\sin ky$

$\Phi(x,a) = 0 \Rightarrow k = \frac{m\pi}{a}$, $m = 1, 2, 3, \cdots$ $\therefore \Phi(x,y) = \sum_{m=1}^{\infty} A_m e^{-\frac{m\pi}{a}x}\sin(\frac{m\pi}{a}y)$

$\Phi(0,y) = 100 = \sum_{m=1}^{\infty} A_m \sin(\frac{m\pi}{a}y)$, $0 \le y \le a$

orthogonality: $\int_0^a 100\sin(\frac{m\pi}{a}y)dy = A_m\int_0^a \sin^2(\frac{m\pi}{a}y)dy = \frac{a}{2}A_m$

$\therefore A_m = \frac{2}{a}\int_0^a 100\sin(\frac{m\pi}{a}y)dy = \frac{200}{m\pi}(1-\cos m\pi) = \begin{cases} 0, & m = 2, 4, 6, \cdots \\ \frac{400}{m\pi}, & m = 1, 3, 5, \cdots \end{cases}$

$\therefore \Phi(x,y) = \frac{400}{\pi}\sum_{m\,\mathrm{odd}}^{\infty}\frac{1}{m}\cdot e^{-\frac{m\pi}{a}x}\sin(\frac{m\pi}{a}y)$

11.25 (a) $\Phi(r) = \frac{C_1}{r} + C_2$ $\Phi(a) = V_0 = \frac{C_1}{a} + C_2$ $\Phi(b) = 0 = \frac{C_1}{b} + C_2$

Solve for C_1 & C_2: $C_1 = \frac{V_0}{\frac{1}{a} - \frac{1}{b}}$ $C_2 = \frac{-V_0}{b(\frac{1}{a} - \frac{1}{b})}$

$\Phi(r) = \frac{V_0}{(\frac{1}{a} - \frac{1}{b})}(\frac{1}{r} - \frac{1}{b})$ for $a < r < b$

(b) $E_r = -\frac{\partial\Phi}{\partial r} = \frac{V_0}{(\frac{1}{a} - \frac{1}{b})}\frac{1}{r^2}$ for $a < r < b$

(c) $D_{r1} = \frac{V_0\varepsilon_1}{(\frac{1}{a} - \frac{1}{b})}\frac{1}{r^2}$ in medium 1

$D_{r2} = \frac{V_0\varepsilon_2}{(\frac{1}{a} - \frac{1}{b})}\frac{1}{r^2}$ in medium 2

(d) $Q_1 = 2\pi a^2 D_{r1}(r=a) = 2\pi\frac{V_0\varepsilon_1}{(\frac{1}{a} - \frac{1}{b})}$

$Q_2 = 2\pi a^2 D_{r2}(r=a) = 2\pi\frac{V_0\varepsilon_2}{(\frac{1}{a} - \frac{1}{b})}$

$Q = Q_1 + Q_2 = \frac{2\pi V_0}{(\frac{1}{a} - \frac{1}{b})}(\varepsilon_1 + \varepsilon_2)$

$C = \frac{Q}{V_0} = \frac{2\pi(\varepsilon_1 + \varepsilon_2)}{(\frac{1}{a} - \frac{1}{b})}$.

Solution: The resistivity read by the sonde will be influenced by medium 2. Thus, the sonde will not read $\rho_r = 10\ \Omega$-m, although it is located entirely in medium 1. To find the expected reading, we must first calculate the potential detected at B. We solved the potential problem in Example 12.5. In the present case, we have $\sigma_1 = 0.1$, $x = 0$, $y = 0$, $z - h = 16$ in. $\times$ (2.54/100) m/in. $= 0.406$ m, $z + h = 32 \times 2.54/100$ m $= 0.813$ m, and

$$I' = I\left(\frac{0.1 - 0.01}{0.1 + 0.01}\right) = 0.818I$$

Therefore, according to (12.22a),

$$\Phi_B = \frac{I}{4\pi \times 0.1}\left[\frac{1}{0.406} + \frac{0.818}{0.813}\right] = \frac{I}{4\pi}(34.7)$$

Substituting the above value into (12.26), we obtain

$$\rho_r = 4\pi \times 16 \times \frac{2.54}{100} \times \frac{1}{4\pi} \times 34.7 = 14.1\ \Omega\text{-m}$$

Problems

12.1 A parallel plate is filled with two materials in a configuration shown in Figure P12.1. The total area of the plate is A. The dielectric constant and the conductivity of one material are ϵ_1 and σ_1, respectively. Those of the other material are ϵ_2 and σ_2. Find the equivalent circuit for this parallel plate, and express the circuit parameters in terms of A, d, ϵ_1, σ_1, ϵ_2, and σ_2.

12.2 A parallel plate is filled with two materials in a configuration shown in Figure P12.2. Find its equivalent circuit, and express the circuit parameters in terms of A, the area of the plate, and d_1, d_2, ϵ_1, ϵ_2, σ_1, and σ_2, which are defined in the figure.

12.3 A coaxial line has two layers of insulation. Figure P12.3 shows the geometry. Find

(a) the potential Φ_1 for $a < \rho < b$
(b) the potential Φ_2 for $b < \rho < c$
(c) the resistance of a section of such a line ℓ meters long

Figure P12.1

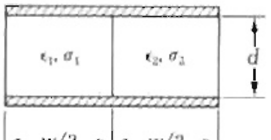


Figure P12.2

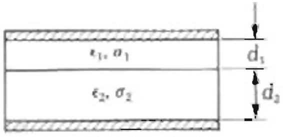
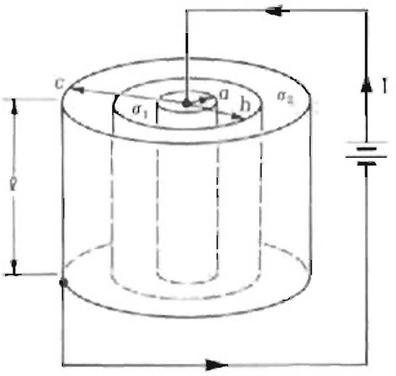


Figure P12.3



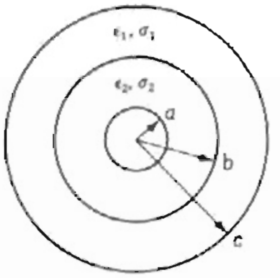


Figure P12.4

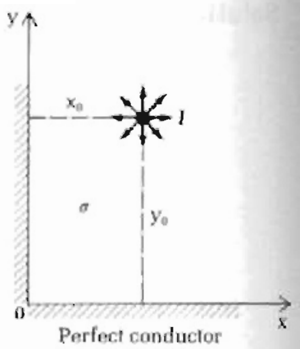


Figure P12.6

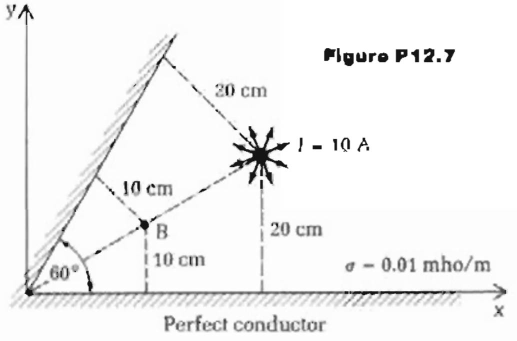


Figure P12.7

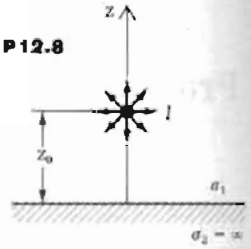


Figure P12.8

12.4 A spherical conductor of radius a is inside a spherical conducting shell of radius c. Two materials are used to fill the space between these conductors. The dielectric constants and the conductivities of these materials are ϵ_1, σ_1, ϵ_2, σ_2, respectively. Figure P12.4 shows the configuration. Find the equivalent circuit of this system, and express the circuit parameters in terms of a, b, c, ϵ_1, ϵ_2, σ_1, and σ_2.

12.5 Two oil wells are 1 km apart. The resistance between two steel pipes in these wells is measured at 3.41 Ω. What is the conductivity of the ground near these wells? Use the following data: the length of both pipes = 1 km, and the diameter of both pipes = 10 cm.

12.6 A current electrode is near a perfectly conducting plate that is bent to form a 90° corner, as shown in Figure P12.6. The output from the electrode is I amperes, and the material filling the space has a conductivity equal to σ. Find the potential function $\Phi(x, y, z)$.

12.7 A current electrode is near a perfectly conducting plate that is bent to form a 60° corner, as shown in Figure P12.7. The electrode produces 10 A of current, and the material filling the region defined by $0 < \phi < 60°$ is water with conductivity equal to 0.01 mho/m. Find the potential at point B shown in the figure.

12.8 A point electrode puts out I amperes of current above a conducting plane, as shown in Figure P12.8.

(a) Find $\Phi(x, y, z)$ for $z > 0$.
(b) Find the current density $J_z(x, y)$ at the surface of the conductor.
(c) Sketch the paths of the current flow.

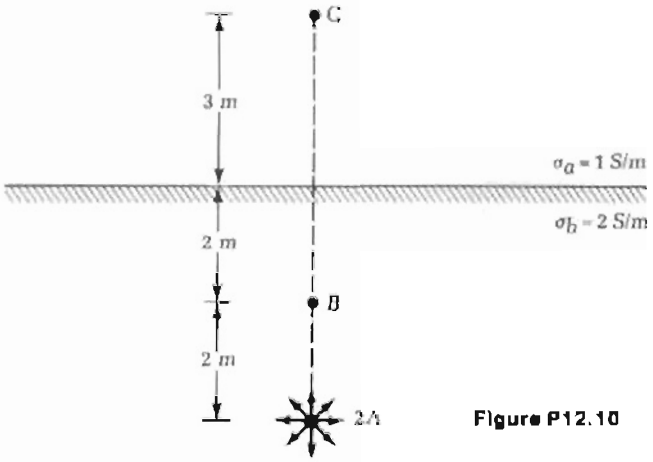


Figure P12.10

12.9 For the case shown in Figure 12.9, find the percentage of the current emitted from the electrode crosses the boundary and enters in medium 2.

12.10 A source 4 meters below an interface of two conducting media emits 2 A of direct current, as shown in Figure P12.10.

(a) Calculate the potential at point B.
(b) Calculate the potential at point C.

12.11 A well-logging resistivity tool similar to the one shown in Figure 12.12 is near a boundary between two beds, as shown in Figure P12.11. The boundary is making a 60° angle with the well. Find the apparent resistivity measured by this tool at the position shown.

12.12 Refer to Example 12.6. Obtain ρ_a (the apparent resistivity measured by the tool) as a function of tool position for $z_0 = +160$ in. to $z_0 = -160$ in., where z_0 is the position of the center of the tool (the midpoint between electrodes A and B) relative to the boundary. Calculate ρ_a for at least 21 points, and plot ρ_a versus z_0.

12.13 Repeat Problem 12.12 for the situation shown in Figure P12.11.

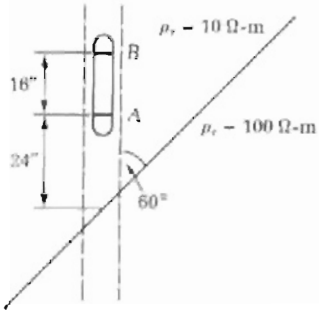
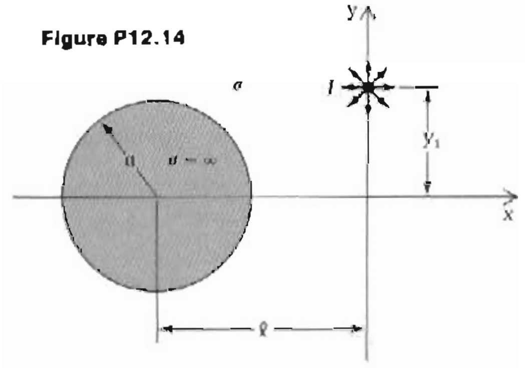


Figure P12.11

12.14 A point electrode is located at $(0, y_1, 0)$, and a perfectly conducting sphere of radius a is located at $(-\ell, 0,0)$ as shown in Figure P12.14. The electrode gives I amperes of current. The conductivity of the medium is σ. Find the potential Φ on the y axis. **Hint:** use (11.44).

12.15 Consider a well-logging resistivity tool similar to the one shown in Figure 12.10. Let the spacing between the current electrode A and the potential electrode B be 6 m. The tool measures the conductivity of the earth formation as it travels in a well. Assume that the well passes near a mineral deposit modeled by a perfectly conducting sphere, as shown in Figure P12.15. Find the apparent resistivity measured by the tool as a function of y. Use the following data: $\sigma = 0.01$ mho/m for the ground; the radius of the mineral deposit = 50 m; and the distance between the center of the sphere and the well = 70 m. Plot σ_{apparent} versus y for $-70 < y < 70$. Hint: use the result obtained in the preceding problem.

Figure P12.14



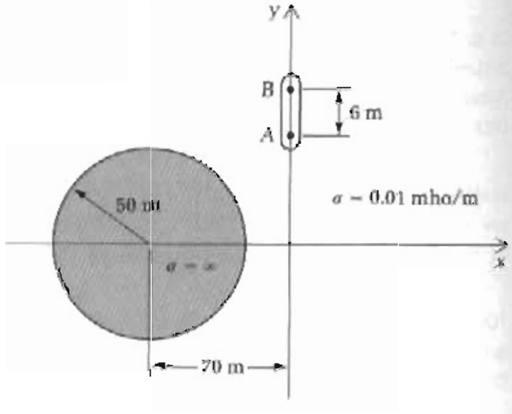
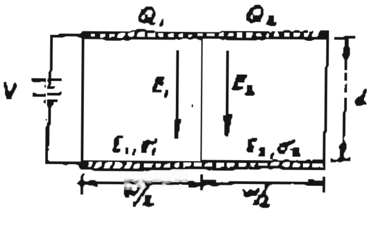
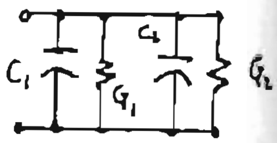


Figure P12.15

CHAPTER 12

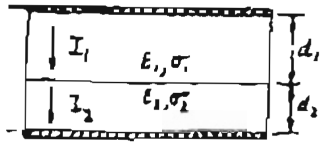
2.1 Because tangential E-field has to be continuous, $E_1 = E_2$. Consequently, $V = E_1 d = E_2 d$ and we can consider it as two imperfect capacitors in parallel (Example 12.1):

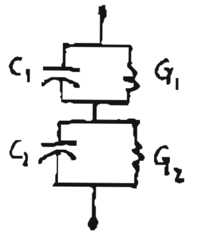




$$C_1 = \frac{A\varepsilon_1}{2d}, \quad C_2 = \frac{A\varepsilon_2}{2d}, \quad G_1 = \frac{A\sigma_1}{2d}, \quad G_2 = \frac{A\sigma_2}{2d}$$

2.2 Because the current has to be continuous, $I_1 = I_2$ and we can consider it as two imperfect capacitors in series.

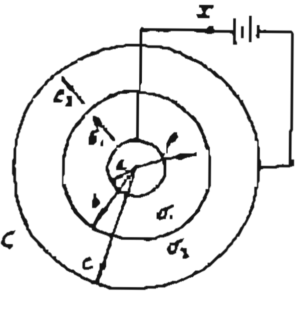




Each capacitor is represented by an ideal C and a conductance G. $C_1 = \frac{A\varepsilon_1}{2d_1}$, $G_1 = \frac{A\sigma_1}{2d_1}$, $C_2 = \frac{A\varepsilon_2}{2d_2}$, $G_2 = \frac{A\sigma_2}{2d_2}$.

12.3 $E_1 = \frac{I}{2\pi\sigma_1 l\rho}$, $a \le \rho \le b$

$E_2 = \frac{I}{2\pi\sigma_2 l\rho}$, $b \le \rho \le c$



(a) $\Phi_1(\rho) = -\int_a^\rho E_1 d\rho = \frac{I}{2\pi\sigma_1 l} \ln(a/\rho)$, $a \le \rho \le b$

(b) $\Phi_2(\rho) = -\int_b^\rho E_2 d\rho - \int_a^b E_1 d\rho = \frac{I}{2\pi\sigma_2 l} \ln(b/\rho) + \frac{I}{2\pi\sigma_1 l} \ln(a/b)$, $b \le \rho \le c$

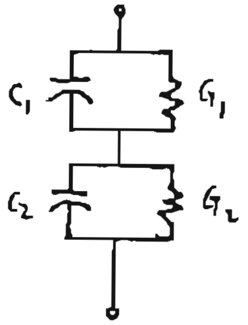
(c) $\Phi(a) - \Phi(c) = \Phi_1(a) - \Phi_2(c) = 0 - \frac{I}{2\pi\sigma_1 l} \ln(a/b) - \frac{I}{2\pi\sigma_2 l} \ln(b/c)$

$$\therefore R = \frac{\Phi(a) - \Phi(c)}{I} = \frac{1}{2\pi l}\left\{ \frac{1}{\sigma_1} \ln(b/a) + \frac{1}{\sigma_2} \ln(c/b) \right\}$$

12.4 The system can be considered as two imperfect capacitors in series. The model for each capacitor is given in Example 12.2.

$$C_1 = \frac{4\pi\varepsilon_1}{\frac{1}{b} - \frac{1}{c}}, \quad G_1 = \frac{4\pi\sigma_1}{\frac{1}{b} - \frac{1}{c}}$$

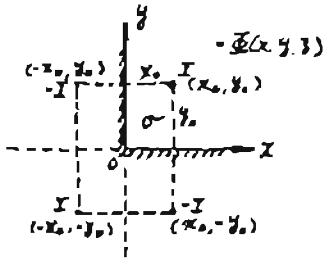
$$C_2 = \frac{4\pi\varepsilon_2}{\frac{1}{a} - \frac{1}{b}}, \quad G_2 = \frac{4\pi\sigma_2}{\frac{1}{a} - \frac{1}{b}}.$$



12.5 $G = \frac{\pi\sigma l}{\cosh^{-1}(h/a)}$; $l = 10^3 m$, $h = 5\times10^2 m$, $a = 5\times10^{-2} m$. $G = \frac{1}{341}$

$\sigma = \frac{\cosh^{-1}(h/a)}{\pi l}\cdot G = \frac{\cosh^{-1}(5\times10^2/5\times10^{-2})}{\pi\times10^3}\times\frac{1}{3.41} = 9.24\times10^{-4}$ mho/m

12.6 use image method:



$\Phi(x,y,z) = \frac{I}{4\pi\sigma}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4}\right)$,

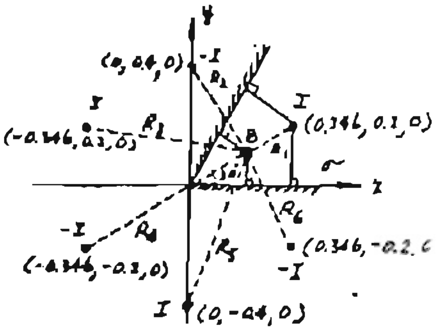
where $R_1 = [(x-x_0)^2 + (y-y_0)^2 + z^2]^{1/2}$

$R_2 = [(x+x_0)^2 + (y-y_0)^2 + z^2]^{1/2}$

$R_3 = [(x+x_0)^2 + (y+y_0)^2 + z^2]^{1/2}$

$R_4 = [(x-x_0)^2 + (y+y_0)^2 + z^2]^{1/2}$

12.7 use image method:



$\Phi_B(x,y,z) = \frac{I}{4\pi\sigma}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4} + \frac{1}{R_5} - \frac{1}{R_6}\right)$,

where $R_1 = [(x-0.346)^2 + (y-0.1)^2 + z^2]^{1/2}$, $R_6 = [(x-0.346)^2 + (y+0.2)^2 + z^2]^{1/2}$

$R_2 = [x^2 + (y-0.4)^2 + z^2]^{1/2}$, $R_5 = [x^2 + (y+0.4)^2 + z^2]^{1/2}$

$R_3 = [(x+0.346)^2 + (y-0.2)^2 + z^2]^{1/2}$, $R_4 = [(x+0.346)^2 + (y+0.2)^2 + z^2]^{1/2}$

at B, $x = 0.173$ m, $y = 0.1$ m, $z = 0$ and

$R_1 = 0.2$ m, $R_2 = 0.346$ m, $R_3 = 0.529$ m, $R_4 = 0.6$ m, $R_5 = 0.529$ m, $R_6 = 0.346$ m

with $\sigma = 0.01$, $I = 10$; $\Phi_B = \frac{10}{4\pi\times0.01}\left(\frac{1}{0.2} - \frac{1}{0.346} + \frac{1}{0.529} - \frac{1}{0.6} + \frac{1}{0.529} - \frac{1}{0.346}\right) = 106$ V

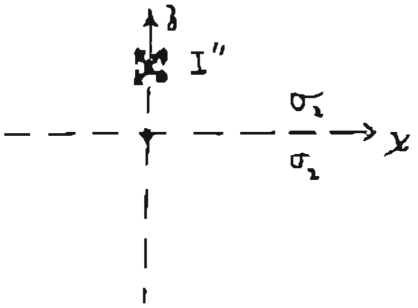
12.8 (a) $\Phi(x,y,z) = \frac{I}{4\pi\sigma_1}\left[\frac{1}{\sqrt{x^2+y^2+(z-z_0)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+z_0)^2}}\right]$

(b) $E_z = -\frac{\partial\Phi}{\partial z} = \frac{I}{4\pi\sigma_1}\left\{\frac{z-z_0}{[x^2+y^2+(z-z_0)^2]^{3/2}} - \frac{z+z_0}{[x^2+y^2+(z+z_0)^2]^{3/2}}\right\}$

$J_z(x,y) = \sigma_1 E_z\big|_{z=0} = -\frac{I}{2\pi}\frac{z_0}{(x^2+y^2+z_0^2)^{3/2}}$

(c) same as Figure 11.7

12.9 For observer in medium 2, the situation looks like this: a source I'' located at $z=h$, in a homogeneous medium of σ_2. Therefore $\frac{1}{2}$ of I'' will cross the x-axis into the lower half of the space.



$$\text{current crossing } x\text{-axis} = \tfrac{1}{2} I'' = \frac{\sigma_2}{\sigma_1+\sigma_2} I$$

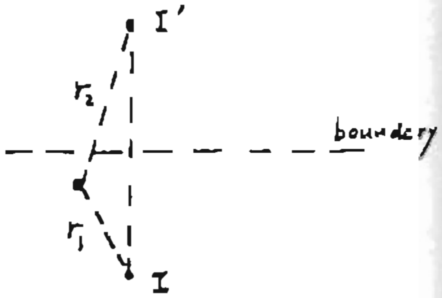
$$\%\text{ of current crossing } x\text{-axis} = [\sigma_2/(\sigma_1+\sigma_2)] \times 100\,\%$$

12.10 (a) In lower medium,

$$\Phi = \frac{I}{4\pi\sigma_b r_1} + \frac{I'}{4\pi\sigma_b r_2}$$

$$I' = \frac{\sigma_b - \sigma_a}{\sigma_b + \sigma_a} I = \frac{2}{3}$$

$$\Phi \text{ at B} = \frac{2}{4\pi\cdot 2\cdot 2} + \frac{2/3}{4\pi\cdot 2\cdot 6} = 44.2 \text{ mV}$$

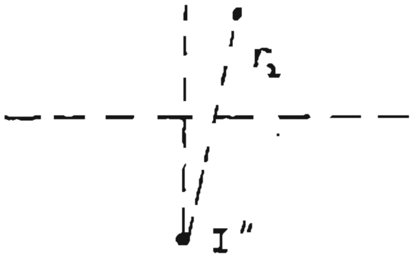


(b) In upper medium,

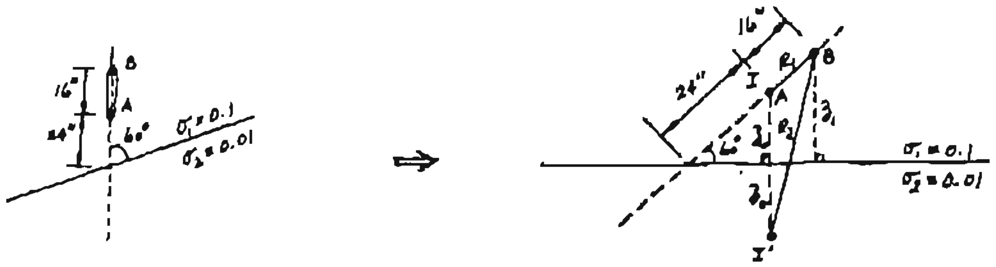
$$\Phi = \frac{I''}{4\pi\sigma_a r_2}$$

$$I'' = \frac{2\sigma_a}{\sigma_a+\sigma_b} I = \frac{4}{3}$$

$$\Phi \text{ at C} = \frac{4/3}{4\pi\cdot 1\cdot 7} = 15.2 \text{ mV}$$



12.11



$$I' = I\left(\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}\right) = 0.818\, I$$

$R_1 = 16" = 0.4064\ m$, $z_0 = 24" \sin 60^\circ = 20.78" = 0.528\ m$, $z_1 = (24"+16")\sin 60^\circ = 34.64" = 0.88\ m$

$$R_2 = \left[(R_1\cos 60^\circ)^2 + (z_0+z_1)^2\right]^{1/2} = 56" = 1.4224\ m$$

$$\Phi_B = \frac{1}{4\pi\sigma_1}\left(\frac{I}{R_1}+\frac{I'}{R_2}\right) = \frac{I}{4\pi\sigma_1}\left(\frac{1}{0.4064}+\frac{0.818}{1.4224}\right) = \frac{I}{4\pi\sigma_1}(3.0357)$$

$$\therefore\ \sigma_{app} = \frac{I}{4\pi R_1 \Phi_B} = \frac{\sigma_1}{R_1(3.0357)} = \frac{0.1}{(0.4064)(3.0357)} = 0.081\ \text{mho/m} \Rightarrow \rho_{app} = \frac{1}{0.081} = 12.34\ \text{ohm-m}$$

12.12 (i) for $z_0 > \ell/2$; A and B in medium 1, $I' = \frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2} I$

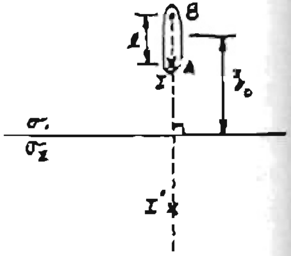
$$\Phi_B = \frac{1}{4\pi\sigma_1}\left[\frac{I}{\ell}+\frac{I'}{2|z_0|}\right] \Rightarrow \rho_a = 4\pi\ell\Phi_B/I = \frac{1}{\sigma_1}\left[1+\left(\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}\right)/2\left(\frac{|z_0|}{\ell}\right)\right]$$

(ii) for $|z_0| < \ell/2$; A in medium 2 and B in medium 1, $I'' = \frac{2\sigma_1}{\sigma_1+\sigma_2} I$

$$\Phi_B = \frac{I''}{4\pi\sigma_1\ell} = \frac{I}{4\pi\ell}\left(\frac{2}{\sigma_1+\sigma_2}\right) \Rightarrow \rho_a = 4\pi\ell\Phi_B/I = \frac{2}{\sigma_1+\sigma_2}$$

(iii) for $z_0 < -\ell/2$; A and B in medium 2, $I' = \frac{\sigma_2-\sigma_1}{\sigma_2+\sigma_1} I$

$$\Phi_B = \frac{1}{4\pi\sigma_2}\left[\frac{I}{\ell}+\frac{I'}{2|z_0|}\right] \Rightarrow \rho_a = 4\pi\ell\Phi_B/I = \frac{1}{\sigma_2}\left[1+\left(\frac{\sigma_2-\sigma_1}{\sigma_2+\sigma_1}\right)/2\left(\frac{|z_0|}{\ell}\right)\right]$$



12.13 (i) for $z_0 > \ell/2$; A and B in medium 1, $I' = \frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2} I$

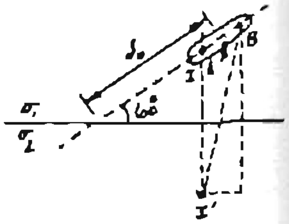
$$\Phi_B = \frac{1}{4\pi\sigma_1}\left\{\frac{I}{\ell}+\frac{I'}{\sqrt{(2z_0\sin 60^\circ)^2+(\ell\cos 60^\circ)^2}}\right\} \Rightarrow \rho_a = 4\pi\ell\Phi_B/I = \frac{1}{\sigma_1}\left\{1+\frac{\left(\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}\right)}{\sqrt{0.25+3(z_0/\ell)^2}}\right\}$$

(ii) for $|z_0| < \ell/2$; A in medium 2 and B in medium 1, $I'' = \frac{2\sigma_1}{\sigma_1+\sigma_2} I$

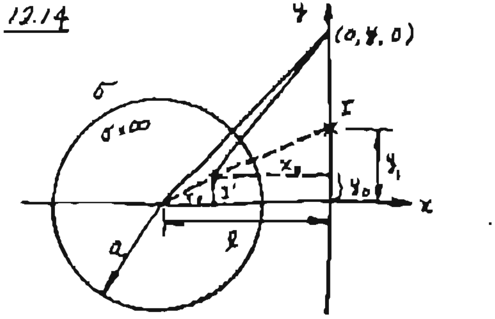
$$\Phi_B = \frac{1}{4\pi\sigma_1}\frac{I''}{\ell} \Rightarrow \rho_a = 4\pi\ell\Phi_B/I = \frac{2}{\sigma_1+\sigma_2}$$

(iii) for $z_0 < -\ell/2$; A and B in medium 2, $I' = \frac{\sigma_2-\sigma_1}{\sigma_2+\sigma_1} I$

$$\Phi_B = \frac{1}{4\pi\sigma_2}\left[\frac{I}{\ell}+\frac{I'}{\sqrt{(2z_0\sin 60^\circ)+\ell^2\cos^2 60^\circ}}\right] \Rightarrow \rho_a = 4\pi\ell\Phi_B/I = \frac{1}{\sigma_2}\left\{1+\frac{\left(\frac{\sigma_2-\sigma_1}{\sigma_2+\sigma_1}\right)}{\sqrt{0.25+3(z_0/\ell)^2}}\right\}$$



12.14

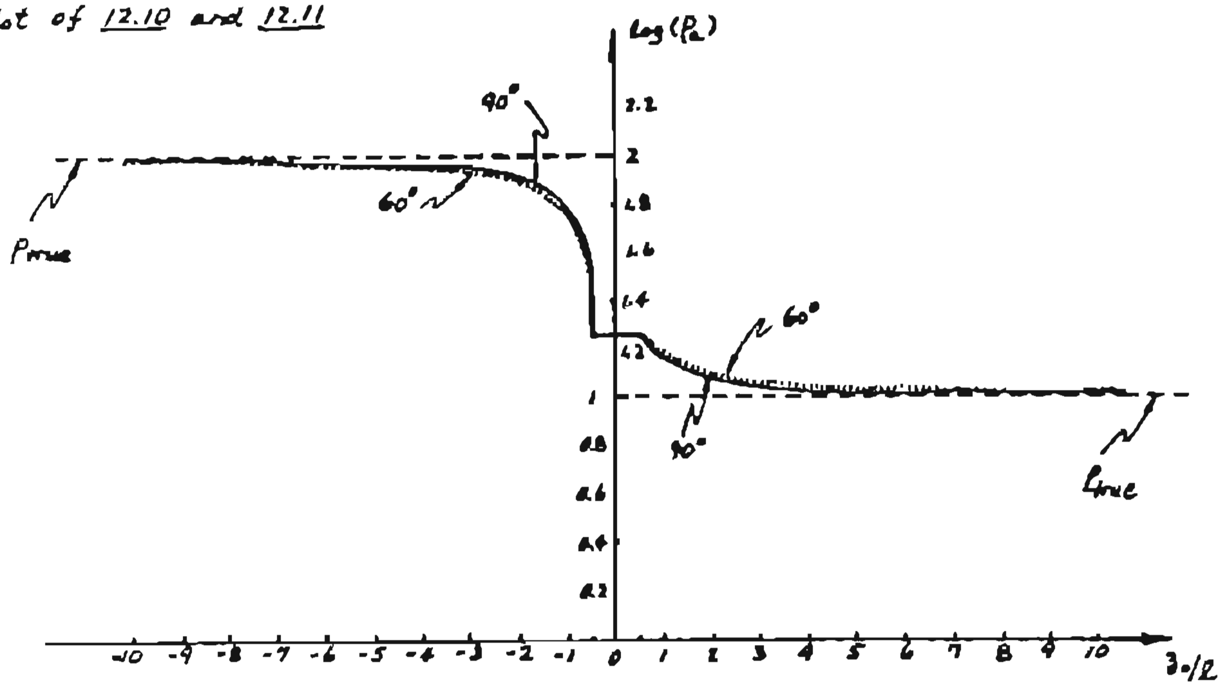


$$r_0 = a^2/\sqrt{y_1^2+\ell^2}\ ;\quad I' = -I\left(a/\sqrt{y_1^2+\ell^2}\right)$$

$$\Phi(y) = \frac{I}{4\pi\sigma}\left\{\frac{1}{|y-y_1|} - \frac{a/\sqrt{y_1^2+\ell^2}}{\sqrt{x_0^2+(y-y_0)^2}} + \frac{a/\sqrt{\ell^2+y_1^2}}{\sqrt{\ell^2+y^2}}\right\}$$

$$= \frac{I}{4\pi\sigma}\left\{\frac{1}{|y-y_1|} - \frac{a/\sqrt{y_1^2+\ell^2}}{\sqrt{\left(\ell-\frac{a^2\ell}{y_1^2+\ell^2}\right)^2+\left(y-\frac{a^2y_1}{y_1^2+\ell^2}\right)^2}} + \frac{a/\sqrt{\ell^2+y_1^2}}{\sqrt{\ell^2+y^2}}\right\}$$

Plot of 12.10 and 12.11



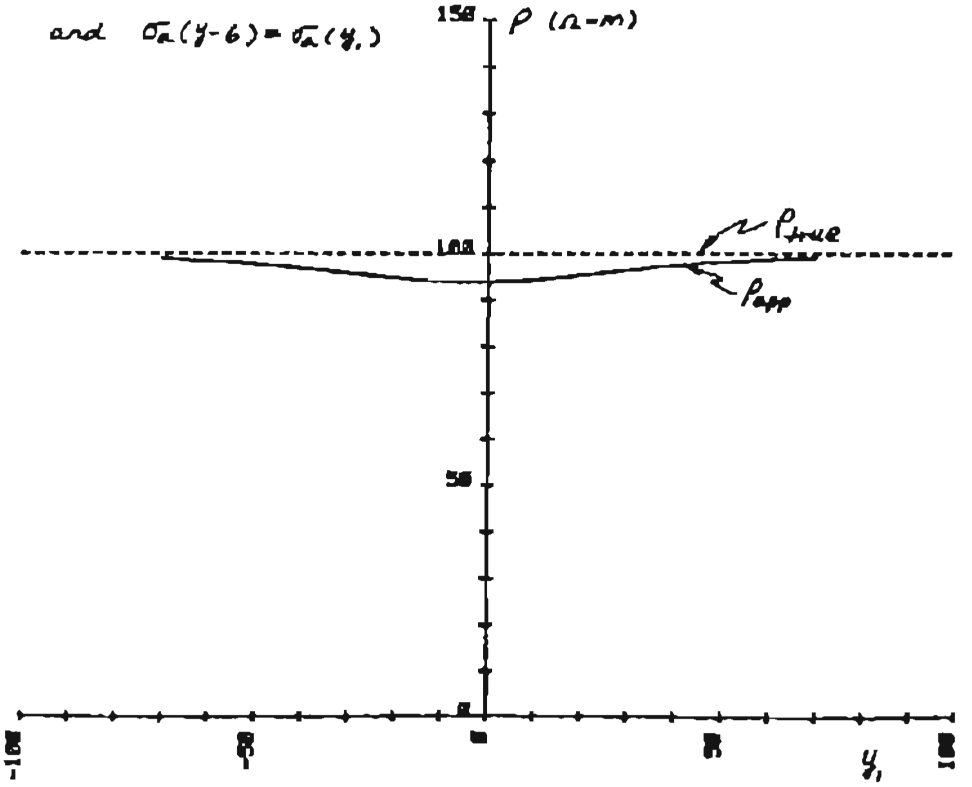
12.15 From Prob 12.12,

$$\rho_a(y_1) = 4\pi|y-y_1|\Phi(y)/I = \frac{|y-y_1|}{\sigma}\left\{\frac{1}{|y-y_1|} - \frac{a}{\sqrt{\frac{\ell^2}{y_1^2+\ell^2}(y_1^2+\ell^2-a^2)^2 + \frac{1}{y_1^2+\ell^2}\left[y(y_1^2+\ell^2)-a^2y_1\right]^2}} + \frac{a}{\sqrt{(\ell^2+y_1^2)(\ell^2+y^2)}}\right\}$$

With $a = 50^m$, $\ell = 70^m$, $\sigma = 0.01$ mho/m, and $|y_1 - y| = 6^m$.

$$\rho_a(y_1) = 600\left\{\frac{1}{6} - \frac{50}{\sqrt{\frac{4900}{y_1^2+4900}(y_1^2+2400)^2 + \frac{1}{y_1^2+4900}\left[(y_1+6)(y_1^2+4900)-2500y_1\right]^2}} + \frac{50}{\sqrt{(4900+y_1^2)(y_1^2+12y_1+4936)}}\right\}$$

$\therefore \sigma_a(y_1) = \frac{1}{\rho_a(y_1)}$ and $\sigma_a(y-6) = \sigma_a(y_1)$



Because magnetic field is present in the coaxial line, we know that magnetic energy is stored there. The magnetic field is given by (13.7):

$$H_\phi = \begin{cases} \dfrac{I}{2\pi\rho} & b > \rho > a \\ 0 & \text{elsewhere} \end{cases}$$

Substituting the above expression in (13.34), we obtain

$$U_H = \frac{1}{2}\mu \int_0^{2\pi} d\phi \int_a^b \rho\, d\rho \frac{I^2}{4\pi^2\rho^2} = \frac{\mu I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

This result is the stored magnetic energy per unit length of the coaxial line. Consequently, we can calculate the inductance per unit length of the line from (13.41):

$$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \qquad (13.49)$$

This inductance per unit length also appears in the transmission-line representation of the coaxial line in (6.19) of Chapter 6.

Problems

13.1 Find the magnetic field H at the center of a square loop carrying a current I. The side of the square loop is b meters long.

13.2 A circular loop that has radius a and that carries a current I produces the same magnetic-field strength at its center as that at the center of a square loop that has side b and that carries the same current I. Find the ratio of b to a.

13.3 Consider a large conducting plate of thickness d located at $-d/2 \le y \le d/2$, as shown in Figure P13.3. Uniform current of density J is flowing in the $\hat{z}$ direction. Find H in all regions.

13.4 The earth's magnetic field at the north magnetic pole is approximately 0.62 G ($1\ \text{G} = 10^{-4}\ \text{Wb/m}^2$). Assume that this magnetic field is produced by a loop of current flowing along the equator. Estimate the magnitude of this current. The radius of the earth is approximately 6,500 km.

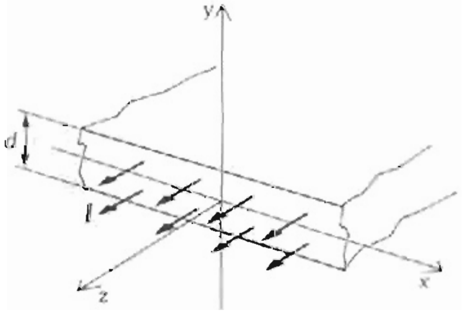


Figure P13.3

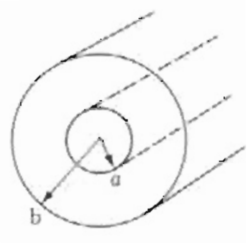


Figure P13.5

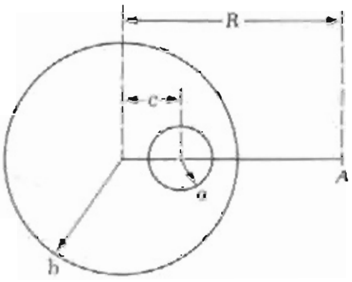


Figure P13.6

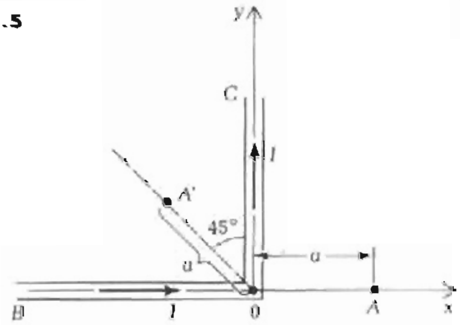


Figure P13.7

13.5 An infinitely long tubular conductor of inner radius a and outer radius b carries a direct current of I amperes, as shown in Figure P13.5. Find the H field at ρ; where (a) $\rho \le a$, (b) $a \le \rho \le b$, and (c) $b \le \rho$.

13.6 An infinitely long tubular conductor has outer radius b and inner radius a offset by a distance c from the axis of the outer cylinder, as shown in Figure P13.6. This eccentric tubular conductor carries a direct current of I amperes. Find the H field at point A shown in the figure. **Hint:** Consider the tube to be a superposition of two solid cylinders that have radii b and a and that carry uniform current density J in opposite directions.

13.7 An infinitely long wire is bent to form a 90° corner, as shown in Figure P13.7. A direct current I flows in the wire. At point A find the H field due to this current. Follow the steps given below.

(a) Use the Biot-Savart law to express the H field at A due to a typical segment of wire dy on the wire axis. Express the field in rectangular coordinates.
(b) Integrate the result obtained in (a) to find the H field due to the semi-infinite wire **OC**. Note: to facilitate integration, let $y = a \tan \theta$, so that $dy = a \sec^2 \theta \, d\theta$.
(c) Find the H field at A due to the semi-infinite wire **BO**.
(d) Add the results obtained in (b) and (c) to yield the total field at A due to the current in the wire **BOC**.

13.8 Follow a similar procedure to the one described in Problem 13.7 to find the H field at point A', as shown in Figure P13.7.

13.9 Consider a circular loop carrying a current I counterclockwise, as shown in Figure 13.11. Plot the magnetic field H_z on the z axis for $-a/2 < z < a/2$. Find the value z_0 in terms of a, such that, if $|z| < z_0$, then H_z is uniform within 10% of the value of H_z at the center of the loop.

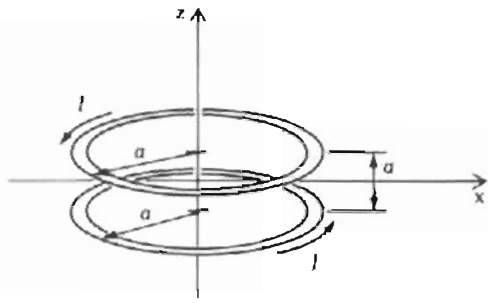


Figure P13.10 Helmholtz coils

13.10 To improve the uniformity of the magnetic field along the axis of a circular loop (see Problem 13.9), one may use two identical loops separated by a distance equal to their radii, as shown in Figure P13.10. Such a pair of current-carrying loops is called **Helmholtz coils.** Find H_z as a function of z on the axis of the Helmholtz coils. Plot H_z for $-a < z < a$. Find, in terms of a, the value z_0 such that, within the range $|z| < z_0$, H_z is uniform within 10% of the magnetic field at the middle of the two coils. Compare your result with that obtained in Problem 13.9 for a single loop.

13.11 A square conductor loop $2a$ meters long on each side carries a direct current I as shown in Figure P13.11.

(a) Calculate the magnetic field $\mathbf{B}$ at $(b,0,0)$. Express the magnetic field in terms of 4 integrals, where each represents the contribution from the current on each side of the square. Use the Biot-Savart law. Do not try to integrate those integrals.

(b) Assume that b is much greater than a. Now, evaluate the integrals approximately to obtain an approximate value of $\mathbf{B}$ at $(b,0,0)$.

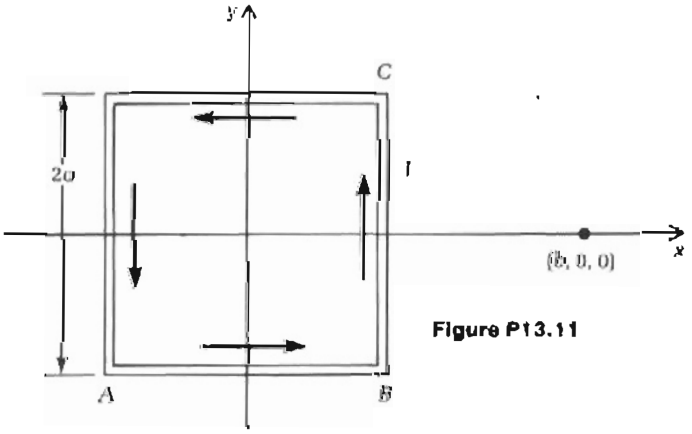


Figure P13.11

13.12 A surface charge of ρ_s C/m^2 is uniformly distributed on a record disk. The inner radius of the disk is a and the outer radius is b. The record disk is turning at a constant angular velocity ω rad/s in the clockwise direction. Find the magnetic field at the center of the disk due to the surface charge on the turning disk. Ignore the presence of the metal post on the turntable.

13.13 The earth's magnetic field at the equator is approximately $B = 10^{-4}$ Wb/m^2. Calculate the cyclotron frequency of the electron in the ionosphere.

13.14 Because natural uranium contains a slight amount of Uranium 234, the electromagnetic isotope separator can also yield ^{234}U. If the radius of the circular path for ^{238}U particles (see Figure 13.14) is equal to 10 m, where should one place collectors for ^{235}U and ^{234}U particles? Express spacings in meters.

13.15 Refer to Figure 13.17. The magnetic field is changed from 5×10^{-4} to 10^{-3} Wb/m^2. All other parameters remain unchanged. Find the following:

(a) the position of the electron at the exit side of the magnetic-field region
(b) the exit angle (the angle between the trajectory and the x axis after the electron has passed through the magnetic field)

13.16 Consider an electron having initial kinetic energy $m_e v_0^2/2$ and entering a region of uniform magnetic field, as depicted in Figure P13.16. This situation is similar to that shown in Figure 13.17, except that the electron in the present case is inclined at an α angle with respect to the x axis.

(a) Show that v_x and v_z of the electron after it enters the magnetic field are given by

$$v_x = v_0 \cos(\omega_c t + \alpha)$$
$$v_z = -v_0 \sin(\omega_c t + \alpha)$$

where $\omega_c = q_e B_0 / m_e$ and $t = 0$ corresponds to the moment the electron enters the magnetic field.

(b) Find the coordinates x and z of the electron at time t. Note that $x = 0$ and $z = 0$ at $t = 0$.

(c) Find the point where the electron leaves the magnetic field. Assume $v_0 = 2 \times 10^7$ m/s, $\alpha = 5°$, $\omega_c = 8.77 \times 10^7$ rad/s, and $d = 4$ cm.

(d) Find the angle between the x axis and the trajectory of the electron after it has left the magnetic field. Sketch the entire trajectory, and compare it with the one shown in Figure 13.17.

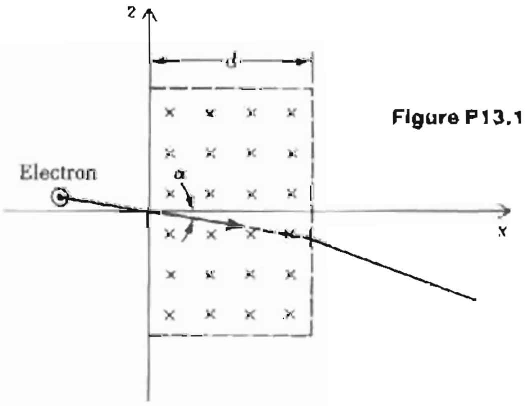


Figure P13.16

13.17 Two parallel wires are carrying 100 A of current in opposite directions. On each wire find the force per unit length due to the magnetic field produced by the other wire. Is the force repulsive or attractive? Assume that the lines are 1.5 m apart.

13.18 Two identical circular loops of radii a are separated by a distance d, where $d \ll a$. One of the coils carries I amperes of current clockwise, and the other carries I amperes counterclockwise. Find the force between these coils. **Hint:** Because these coils are close together, you can approximate the magnetic field that is at one coil and is produced by the current on the other as $H_1 = I_2/(2\pi d)$, the field due to an infinitely long wire. Let $a = 1$ m and $d = 0.05$ m. How much current is needed to produce a force of 9.8 N?

13.19 A circular loop of radius 0.5 m and 100 turns is excited by a 2 A direct current. This loop is placed in the Earth's magnetic field, which is approximately equal to 5×10^{-5} Wb/m^2 pointing north. How do you orient this loop to produce a maximum torque? What is the value of this torque? Find that orientation of the loop in which it experiences no torque.

13.20 The square conducting loop ABCD shown in Figure P13.20 carries 2 A of direct current. Each side of the loop is 0.1 m long. The loop is placed in a uniform magnetic field **B**. Find the force on each side of the loop and the torque on the entire loop if:

(a) $\mathbf{B} = \hat{x}\,0.2$ Wb/m^2
(b) $\mathbf{B} = -\hat{z}\,0.2$ Wb/m^2

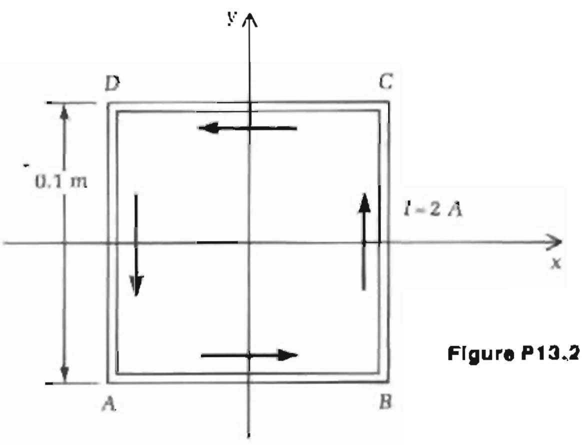


Figure P13.20

13.21 An infinitely long conductor of radius a carries a direction current I as shown in Figure P13.21.

(a) Find the **H** field in the region $0 < \rho < a$.
(b) Calculate the stored magnetic energy per unit length in the region $0 < \rho < a$.
(c) Find the inductance per unit length of the conductor. Consider only the magnetic energy in the region $0 < \rho < a$.

13.22 Three infinitely long parallel wires each carry 10 A of current in the $\hat{z}$ direction, as shown in Figure P13.22. Find the force per unit length acting on the #3 wire due to the magnetic fields produced by the other two wires. Give the numerical value of the force, its direction, and its unit.

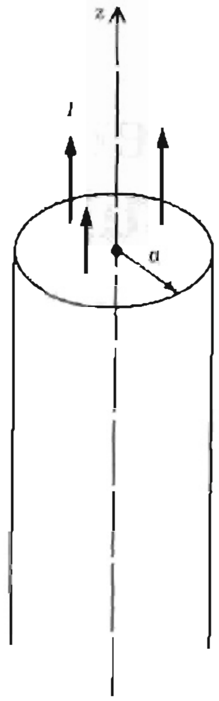


Figure P13.21

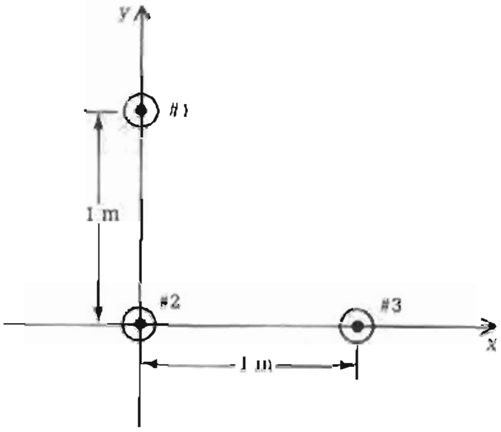


Figure P13.22

13.23 The magnetic field in a coaxial line is given by

$$H_\phi = \begin{cases} 1/\rho & \text{for } 0.1\,\text{m} < \rho < 0.2\,\text{m} \\ 0 & \text{elsewhere} \end{cases}$$

The medium is air. What is the total stored magnetic energy per unit length in the line? Give the numerical value and indicate its unit.

13.24 (a) Calculate the stored magnetic energy per unit length of the parallel-plate conductors shown in Figure 13.5.

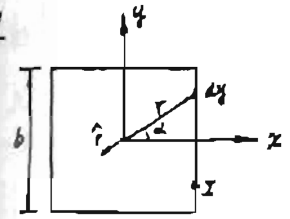
(b) If the parallel plate is used as a capacitor to store electric energy, find the voltage V_0 for which the stored electric energy is equal to the stored magnetic energy found in (a). Let $I = 1\text{A}$, $w = 10$ cm, and $a = 1$ cm. Express V_0 in volts. The medium is air.

13.25 Calculate the inductance per unit length of the coaxial line shown in Figure 13.3a.

13.26 Calculate the inductance per unit length of the parallel-plate conductors shown in Figure 13.5.

CHAPTER 13

13.1



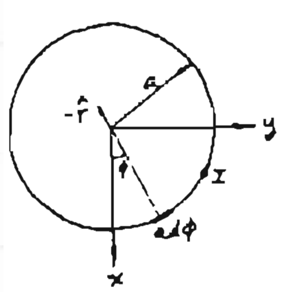
$$d\bar{H}=\frac{I\,dy\,\hat{y}\times\hat{r}}{4\pi r^2}=\frac{I\,dy}{4\pi}\cdot\frac{\hat{y}\times(-\cos\alpha\,\hat{x}-\sin\alpha\,\hat{y})}{y^2+(b/2)^2}=\hat{z}\frac{I}{4\pi}\cdot\frac{\cos\alpha\,dy}{y^2+(b/2)^2}$$
$$=\hat{z}\frac{I}{4\pi}\left(\frac{b}{2}\right)\frac{dy}{[y^2+(b/2)^2]^{3/2}}$$

H field due to one side of the square loop:

$$\bar{H}=\int_{-\frac{b}{2}}^{\frac{b}{2}}d\bar{H}=\hat{z}\frac{Ib}{8\pi}\int_{-\frac{b}{2}}^{\frac{b}{2}}\frac{dy}{[y^2+(b/2)^2]^{3/2}}=\hat{z}\frac{\sqrt{2}I}{2\pi b}$$

$$\therefore \text{Total H field}=4\times\left(\hat{z}\frac{\sqrt{2}I}{2\pi b}\right)=\hat{z}\,2\sqrt{2}\,\frac{I}{\pi b}$$

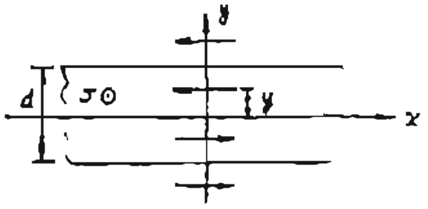
13.2



$$d\bar{H}=\frac{I(a\,d\phi)\,\hat{\phi}\times(-\hat{r})}{4\pi a^2}=\hat{z}\frac{I\,d\phi}{4\pi a}$$
$$\bar{H}=\int d\bar{H}=\hat{z}\frac{I}{4\pi a}2\pi=\hat{z}\frac{I}{2a}$$

Suppose H field due to square loop as in Problem 13.1 is equal to that due to a circular loop as in this problem, then $\frac{2\sqrt{2}I}{\pi b}=\frac{I}{2a}\Rightarrow b/a=\frac{4\sqrt{2}}{\pi}=1.8$

13.3



For $|y|\le d/2$; $2H_x=2yJ$

$\therefore \bar{H}=(-\hat{x})Jy,\ |y|\le d/2$

For $|y|\ge d/2$; $2H_x=2(d/2)J\Rightarrow H_x=J(d/2)$

$$\therefore \bar{H}=\begin{cases}(-\hat{x})J(d/2), & y\ge d/2\\ \hat{x}J(d/2), & y\le -d/2\end{cases}$$

13.4 use equation (13.20) with $z=a=6500\text{ km}=6.5\times10^6\text{ m}$,

$$|\bar{B}|=0.62\times10^{-4}=\frac{\mu I a^2}{2(a^2+a^2)^{3/2}}=\frac{\mu I}{4\sqrt{2}a}\Rightarrow I=(0.62\times10^{-4})\times4\sqrt{2}\times a/\mu=\frac{0.62\times10^{-4}\times4\sqrt{2}\times6.5\times10^6}{4\pi\times10^{-7}}\approx1.81\times10^9$$

13.5 (a) $\rho<a$; $H=0$ (no current enclosed)

(b) $a\le\rho\le b$; $J=\frac{I}{\pi(b^2-a^2)}$ and $2\pi\rho H_\phi=J\cdot\pi(\rho^2-a^2)\Rightarrow\bar{H}=\hat{\phi}\frac{I}{2\pi\rho}\cdot\frac{\rho^2-a^2}{b^2-a^2},\ a\le\rho\le b$

(c) $b\le\rho$; $2\pi\rho H_\phi=I\Rightarrow\bar{H}=\hat{\phi}\frac{I}{2\pi\rho},\ b\le\rho$

13.6

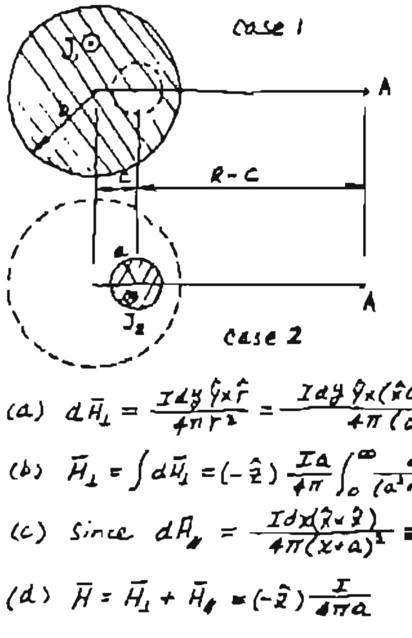


Figure P13.6 is the superposition of case 1 and case 2

Case 1: $J_1=\frac{I}{\pi(b^2-a^2)}$ and $H_\phi^1=\frac{J_1(\pi b^2)}{2\pi R}=\frac{I}{2\pi R}\frac{b^2}{b^2-a^2}$

Case 2: $J_2=\frac{-I}{\pi(b^2-a^2)}$ and $H_\phi^2=\frac{J_2(\pi a^2)}{2\pi(R-C)}=-\frac{I}{2\pi(R-C)}\frac{a^2}{b^2-a^2}$

Total H field $H_\phi=H_\phi^1+H_\phi^2$
$$=\frac{I}{2\pi(b^2-a^2)}\left[\frac{b^2}{R}-\frac{a^2}{R-C}\right]$$

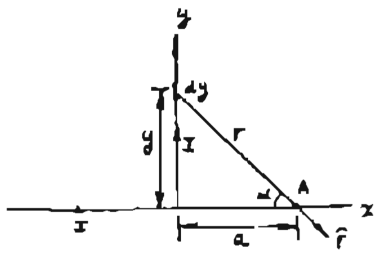
$$\therefore \text{At }A,\ \bar{H}=\hat{\phi}\frac{I}{2\pi(b^2-a^2)}\left[\frac{b^2}{R}-\frac{a^2}{R-C}\right]$$

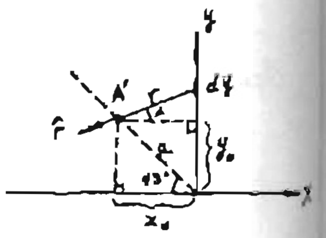
13.7 (a) $d\bar{H}_\perp=\frac{I\,dy\,\hat{y}\times\hat{r}}{4\pi r^2}=\frac{I\,dy\,\hat{y}\times(\hat{x}\cos\alpha-\hat{y}\sin\alpha)}{4\pi(a^2+y^2)}=(-\hat{z})\frac{Ia}{4\pi}\cdot\frac{dy}{(a^2+y^2)^{3/2}}$

(b) $\bar{H}_\perp=\int d\bar{H}_\perp=(-\hat{z})\frac{Ia}{4\pi}\int_0^\infty\frac{dy}{(a^2+y^2)^{3/2}}=(-\hat{z})\frac{I}{4\pi a}\int_0^{\pi/2}\cos\alpha\,d\alpha=(-\hat{z})\frac{I}{4\pi a}$

(c) Since $d\bar{H}_{/\!/}=\frac{I\,dx(\hat{x}\times\hat{x})}{4\pi(x+a)^2}=0$, $\bar{H}_{/\!/}=0$

(d) $\bar{H}=\bar{H}_\perp+\bar{H}_{/\!/}=(-\hat{z})\frac{I}{4\pi a}$





13.8 (a) $d\bar{H}_1 = \frac{I\,dy\,\hat{y}\times\hat{r}}{4\pi r^2} = \frac{I\,dy\,\hat{y}\times(-\cos\alpha\,\hat{x} - \sin\alpha\,\hat{y})}{4\pi[x_0^2+(y-y_0)^2]} = \hat{z}\,\frac{I}{4\pi}\,\frac{x_0\,dy}{[x^2+(y-y_0)^2]^{3/2}}$

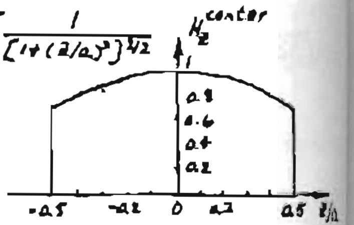
(b) $\bar{H}_1 = \int d\bar{H}_1 = \hat{z}\,\frac{I x_0}{4\pi}\int_0^{\infty}\frac{dy}{[x_0^2+(y-y_0)^2]^{3/2}}$ $\left(\begin{array}{l} y = y_0 + x_0\tan\alpha \\ dy = x_0\sec^2\alpha\,d\alpha \end{array}\right)$

$= \hat{z}\,\frac{I x_0}{4\pi}\cdot\frac{1}{x_0^2}\int_{-\pi/4}^{\pi/2}\cos\alpha\,d\alpha = \hat{z}\,\frac{I}{4\pi x_0}\left(1+\frac{\sqrt{2}}{2}\right)$

$x_0 = a\cos 45^\circ = \frac{\sqrt{2}}{2}a \Rightarrow \bar{H}_1 = \hat{z}\,\frac{I}{4\pi a}\cdot\left(1+\frac{\sqrt{2}}{2}\right)/\frac{\sqrt{2}}{2} = \hat{z}\,\frac{I}{4\pi a}(1+\sqrt{2}) = 0.192\,\frac{I}{a}\,\hat{z}$

(C) due to symmetry, $\bar{H}_y = \bar{H}_1 = 0.192\,\frac{I}{a}\,\hat{z}$

(d) $\bar{H} = \bar{H}_y + \bar{H}_1 = 0.384\,\frac{I}{a}\,\hat{z}$



13.9 From equation (13.20), $H_z = \frac{Ia^2}{2(a^2+z^2)^{3/2}} = \frac{I}{2a}\cdot\frac{1}{[1+(z/a)^2]^{3/2}} = H_z^{center}\cdot\frac{1}{[1+(z/a)^2]^{3/2}}$

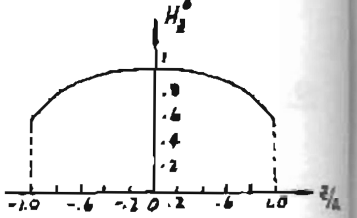
at $z = z_0$, $H_z/H_z^{center} = [1+(z_0/a)^2]^{-3/2} = 0.9$ $\left(H_z^{center} = H_z|_{z=0} = \frac{I}{2a}\right)$

$\Rightarrow -(3/2)\ln[1+(z_0/a)^2] = \ln 0.9$

$\Rightarrow \ln[1+(z_0/a)^2] = 0.07 \Rightarrow (z_0/a)^2 = 0.0728 \Rightarrow z_0 = \pm 0.27a$

13.10 Superposition : $H_z = \frac{Ia^2}{2}\left\{[a^2+(z-\frac{a}{2})^2]^{-3/2} + [a^2+(z+\frac{a}{2})^2]^{-3/2}\right]$

$= \frac{I}{2a}\left\{[1+(\frac{z}{a}-0.5)^2]^{-3/2} + [1+(\frac{z}{a}+0.5)^2]^{-3/2}\right\}$



at $z=0$, $H_z^0 = \frac{I}{2a}\cdot 2\cdot(1+0.25)^{-3/2} = (0.7155)\,\frac{I}{a}$

at $z = z_0$, $H_z = \frac{I}{2a}\left\{[1+(\frac{z_0}{a}-0.5)^2]^{-3/2} + [1+(\frac{z_0}{a}+0.5)^2]^{-3/2}\right\}$

$= \frac{H_z^0}{2\cdot(0.7155)}\left\{[1+(\frac{z_0}{a}-0.5)^2]^{-3/2} + [1+(\frac{z_0}{a}+0.5)^2]^{-3/2}\right\}$

$\frac{H_z}{H_z^0} = \frac{1}{1.431}\left\{[1+(\frac{z_0}{a}-0.5)^2]^{-3/2} + [1+(\frac{z_0}{a}+0.5)^2]^{-3/2}\right\} = 0.9$

$\therefore f(x) = [1+(x-0.5)^2]^{-3/2} + [1+(x+0.5)^2]^{-3/2} - 1.288 = 0$, where $x = \frac{z_0}{a}$

$\Rightarrow x = \frac{z_0}{a} = \pm 0.602$ (range of z_0 is larger than previous problem)

13.11 (a) on $\overline{AB}$, $\bar{dl} = dx\,\hat{x}$, $\hat{r}_{12} = \frac{a\hat{y} + (b-x)\hat{x}}{[a^2+(b-x)^2]^{1/2}}$, $r_{12}^2 = a^2 + (b-x)^2$

$d\bar{H} = \hat{z}\,\frac{Ia}{4\pi}\,dx\,\frac{1}{[a^2+(b-x)^2]^{3/2}}$

$\bar{H}_{AB} = \hat{z}\,\frac{Ia}{4\pi}\int_{-a}^{a}\frac{dx}{[a^2+(b-x)^2]^{3/2}}$

on $\overline{BC}$. $\hat{r}_{12} = -y\hat{y} + (b-a)\hat{x}$, $\bar{dl} = y\,dy$, $r_{12}^2 = y^2 + (b-a)^2$

$\bar{H}_{BC} = \hat{z}\,\frac{-I(b-a)}{4\pi}\int_{-a}^{a}\frac{dy}{[y^2+(b-a)^2]^{3/2}}$

on $\overline{CD}$, $\hat{r}_{12} = \frac{-a\hat{y} + (b-x)\hat{x}}{[a^2+(b-x)^2]^{1/2}}$, $\bar{dl} = -dx\,\hat{x}$, $r_{12}^2 = a^2 + (b-x)^2$

$\bar{H}_{CD} = \hat{z}\,\frac{Ia}{4\pi}\int_{-a}^{a}\frac{dx}{(a^2+(b-x)^2)^{3/2}}$

on $\overline{DA}$, $\hat{r}_{12} = -y\hat{y} + (b+a)\hat{x}$, $r_{12}^2 = y^2 + (b+a)^2$, $\bar{dl} = -dy\,\hat{y}$

$\bar{H}_{DA} = \hat{z}\,\frac{I(b+a)}{4\pi}\int_{-a}^{a}\frac{dy}{[y^2+(b+a)^2]^{1/2}}$

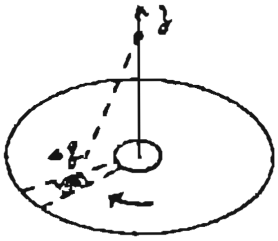
$\bar{H} = \hat{z}\,\frac{I}{4\pi}\left[2\int_{-a}^{a}\frac{a\,dx}{[a^2+(b-x)^2]^{3/2}} + \int_{-a}^{a}\frac{(b+a)\,dy}{[y^2+(b+a)^2]^{3/2}} - \int_{-a}^{a}\frac{(b-a)\,dy}{[y^2+(b-a)^2]^{3/2}}\right]$

(b) $\bar{H} \simeq \hat{z}\,\frac{I}{4\pi}\left[2\int_{-a}^{a}\frac{a\,dx}{b^3} + (b+a)\int_{-a}^{a}\frac{dy}{b^3} - \int_{-a}^{a}\frac{(b-a)\,dy}{b^3}\right]$

$= \hat{z}\,\frac{Ia^2}{\pi b^3}$.

13.12 $\Delta q = \rho_s(\rho\, d\phi\, d\rho)$

$\bar{v} = \rho\omega(-\hat{\phi})$; $\hat{r}_{12} = (-\rho\hat{\rho} + z\hat{z})/\sqrt{\rho^2+z^2}$



$I\, d\bar{\ell} = \Delta q\, \bar{v} = \rho_s(\rho\, d\phi\, d\rho)\rho\omega(-\hat{\phi})$

$$d\bar{H} = \frac{\rho_s \omega \rho^2 d\rho\, d\phi(-\hat{\phi})\times[-\hat{\rho}\rho + z\hat{z}]}{4\pi(\rho^2+z^2)^{3/2}}$$

$$dH_z = \frac{-\rho_s\omega\rho^3 d\rho\, d\phi}{4\pi(\rho^2+z^2)^{3/2}}$$

$\int_0^{2\pi}\hat{\rho}\, d\phi = 0 \rightarrow H_\rho = 0$

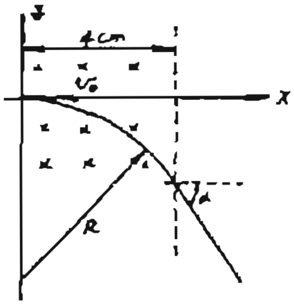
$$H_z = \frac{-\rho_s\omega}{2}\int_a^b \frac{\rho^3 d\rho}{(\rho^2 z^2)^{3/2}}\ .$$

13.13 $f_c = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times10^{-4}}{2\times\pi\times9.11\times10^{-31}} = 2.8\ MHz$

13.14 $R_5/R_8 = (m_5/m_8)^{1/2} = (235/238)^{1/2} = 0.9937 \Rightarrow R_5 = 9.937\, m$

$R_4/R_8 = (m_4/m_8)^{1/2} = (234/238)^{1/2} = 0.99156 \Rightarrow R_4 = 9.916\, m$

13.15 $R = \frac{mv_0}{qB_0} = \frac{9.11\times10^{-31}\times2\times10^7}{1.6\times10^{-19}\times10^{-3}} = 0.114\, m$



(a) $v_x = v_0\cos\omega t$, $v_z = -v_0\sin\omega t$, where $\omega = v_0/R$

$\therefore x = \int_0^t v_x\, dt = \frac{v_0}{\omega}\sin\omega t\Big|_0^t = \frac{v_0}{\omega}\sin\omega t$

$z = \int_0^t v_z\, dt = \frac{v_0}{\omega}\cos\omega t\Big|_0^t = \frac{v_0}{\omega}(\cos\omega t - 1)$

at the exist, $x(t_0) = 0.04 = \frac{v_0}{\omega}\sin\omega t_0 \Rightarrow \sin\omega t_0 = \frac{0.04}{R} = 0.35$

$z(t_0) = \frac{v_0}{\omega}\left[\sqrt{1-\sin^2\omega t_0} - 1\right] = R(\sqrt{1-(0.35)^2} - 1) = -0.0072\, m$

(b) at the exist, $t = t_0$ and $\sin\omega t_0 = 0.35$

$v_x(t_0) = v_0\cos\omega t_0 = v_0\sqrt{1-\sin^2\omega t_0} = v_0\sqrt{1-(0.35)^2} = 0.937\, v_0$

$v_z(t_0) = -v_0\sin\omega t_0 = -0.35\, v_0$

$\alpha = \tan^{-1}[v_z(t_0)/v_x(t_0)] = \tan^{-1}(-0.35/0.937) = -20.5°$

13.16 (a) In the magnetic field region, $v_x^2(t) + v_z^2(t) = v_0^2 \Rightarrow v_x(t) = v_0\cos(\omega_c t+\phi)$ and $v_z(t) = v_0\sin(\omega_c t+\phi$

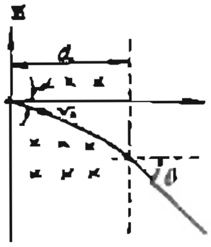
Since $v_x(0) = v_0\cos\alpha$, $v_z(0) = -v_0\sin\alpha \Rightarrow v_x(t) = v_0\cos(\omega_c t+\alpha)$ and $v_z(t) = -v_0\sin(\omega_c t+\alpha)$

(b) In the magnetic field region, $x(t) = \int_0^t v_x(t)\, dt$ and $z(t) = \int_0^t v_z(t)\, dt$

$\therefore x(t) = \frac{v_0}{\omega_c}\sin(\omega_c t+\alpha) + C_1$ and $z(t) = \frac{v_0}{\omega_c}\cos(\omega_c t+\alpha) + C_2$

Since $x(0) = z(0) = 0$, $C_1 = -\frac{v_0}{\omega_c}\sin\alpha$ and $C_2 = -\frac{v_0}{\omega_c}\cos\alpha$

$\therefore x(t) = \frac{v_0}{\omega_c}[\sin(\omega_c t+\alpha) - \sin\alpha]$ and $z(t) = \frac{v_0}{\omega_c}[\cos(\omega_c t+\alpha) - \cos\alpha]$



(c) at the exist, $t = t_0$ and $x(t_0) = d = \frac{v_0}{\omega_c}[\sin(\omega_c t_0+\alpha) - \sin\alpha]$

$\therefore \sin(\omega_c t_0+\alpha) = \frac{\omega_c}{v_0}d + \sin\alpha = \frac{8.77\times10^7\times4\times10^{-2}}{2\times10^7} + \sin(5°) = 0.2626$

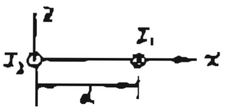
$\therefore \cos(\omega_c t_0+\alpha) = \sqrt{1-\sin^2(\omega_c t_0+\alpha)} = \sqrt{1-(0.2626)^2} = 0.965$

$\therefore$ at the exist, $x(t_0) = 0.04\, m$ and $z(t_0) = \frac{2\times10^7}{8.77\times10^7}[0.965 - \cos(5°)] = -0.00714\, m$

(d) at the exist, $v_x(t_0) = v_0\cos(\omega_c t_0+\alpha) = 0.965\, v_0$ and $v_z(t_0) = -0.2626\, v_0$

exist angle $\theta = \tan^{-1}[v_z(t_0)/v_x(t_0)] = \tan^{-1}(-0.2626/0.965) = -15.22°$

13.17

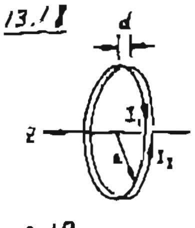


$\bar{H}_{21} = -\hat{z}\frac{I_2}{2\pi d} \Rightarrow \bar{B}_{21} = -\hat{z}\frac{\mu_0 I_2}{2\pi d}$

$d\bar{F}_1 = I_1 dy\, \hat{y} \times \bar{B}_{21} = -\hat{x}\frac{\mu_0 I_1 I_2}{2\pi d} dy \qquad \therefore f_{1x} = \left|\frac{d\bar{F}_1}{dy}\right| = \frac{\mu_0 I_1 I_2}{2\pi d}$

$f_{1x} = \frac{4\pi\times10^{-7}\times(100)^2}{2\pi\times(1.5)} = 1.33\times10^{-3}$ N/m (repulsive)

13.18



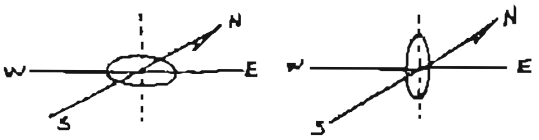
Since $d \ll a$, $H_{21} = \frac{I_2}{2\pi d}$ and $B_{21} = \frac{\mu I_2}{2\pi d}$

$d\bar{F}_1 = I_1 a\, d\phi(-\hat{\phi}) \times \left(\hat{\rho}\frac{\mu I_2}{2\pi d}\right) = \hat{z}\frac{\mu a I_1 I_2}{2\pi d} d\phi \Rightarrow \bar{F}_1 = \int d\bar{F}_1 = \hat{z}\frac{\mu a I_1 I_2}{2\pi d}\int_0^{2\pi} d\phi = \hat{z}\frac{\mu a I_1 I_2}{d}$

For $I_1 = I_2 = I$, $|\bar{F}_1| = 9.8$ N, $a = 1$ m and $d = 0.05$ m,

$I = \left(\frac{|\bar{F}_1| d}{\mu a}\right)^{1/2} = \sqrt{\frac{9.8\times0.05}{4\pi\times10^{-7}\times1}} = 624.4$ A

13.19



Loop should be placed horizontally or Vertically in the E-W direction to obtain the maximum torque.

$T_{max} = mB = \pi(0.5)^2\times100\times2\times5\times10^{-5} = 7.85\times10^{-3}$ N-m

No torque experienced when Loop is placed vertically in the N-S direction

13.20 (a) $\bar{F}_{AB} = 0$, $\bar{F}_{BC} = I d\bar{l}\times\bar{B} = 2\times0.1\times\hat{y}\times0.2\hat{x} = 0.04$ N $(-\hat{z})$; $\bar{F}_{CD} = 0$

$\bar{F}_{DA} = 0.04$ N $(\hat{z})$; $\bar{T} = (0.1)^2\times2\hat{z}\times0.2\hat{x} = 4\times10^{-3}$ N-m $(\hat{y})$

(b) $\bar{F}_{AB} = 0.1\times2\hat{x}\times(-0.2\hat{z}) = 0.04$ N $\hat{y}$; $\bar{F}_{BC} = -0.04$ N $\hat{x}$; $\bar{F}_{CD} = -0.04$ N $\hat{y}$

$\bar{F}_{DA} = 0.04$ N $\hat{x}$; $\bar{T} = 0$.

13.21 (a) $2\pi\rho H_\phi = I\cdot\left(\frac{\pi\rho^2}{\pi a^2}\right) \rightarrow H_\phi = \frac{I\rho}{2\pi a^2}$ for $0<\rho<a$

(b) $U_H = \iiint \frac{1}{2}\mu H_\phi^2\, dv = \frac{1}{2}\mu\frac{I^2}{4\pi^2 a^4}\int_0^{2\pi} d\phi\int_0^a \rho^2(\rho\, d\rho) = \frac{\mu I^2}{16\pi}$

(c) $U_H = \frac{1}{2}LI^2 \qquad L = \frac{2U_H}{I^2} = \frac{2}{I^2}\frac{\mu I^2}{16\pi} = \frac{\mu}{8\pi} = 5\times10^{-8}$ H/m

13.22 $\bar{B}_1 = \frac{10\mu}{2\pi\sqrt{2}}\frac{\hat{x}+\hat{y}}{\sqrt{2}}$

$\bar{B}_2 = \frac{10\mu}{2\pi}\hat{y}$, $\bar{F}_3 = 10\hat{z}\times\left[-\frac{10\mu}{2\pi}\left(\frac{\hat{x}+\hat{y}}{2}+\hat{y}\right)\right] = \frac{25\mu}{\pi}(\hat{y}-3\hat{x}) = 10^{-5}(\hat{y}-3\hat{x})$ N/m

13.23 $U_H = \frac{1}{2}\mu\int_0^{2\pi} d\phi\int_a^b \rho\, d\rho\left(\frac{1}{\rho}\right)^2 = \pi\mu\ln 2 = 2.74\times10^{-6}$ J/m

13.24 (a) $H = I/w \Rightarrow U_H = \frac{1}{2}\mu_0 (I/w)^2 \cdot (aw) = \frac{1}{2}\mu_0 I^2 \frac{a}{w}$

(b) $E = V_0/a \Rightarrow U_E = \frac{1}{2}\varepsilon_0 (V_0/a)^2 \cdot (aw) = \frac{1}{2}\varepsilon_0 V_0^2 \frac{w}{a}$

if $U_H = U_E$, then $\frac{1}{2}\mu_0 I^2 \frac{a}{w} = \frac{1}{2}\varepsilon_0 V_0^2 \frac{w}{a} \Rightarrow V_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{a}{w}\right) I = 120\pi \times 0.1 \times 1 = 37.7\ \mathrm{V}$

13.25 For $0 < \rho < a$, $U_{H1} = \frac{\mu_0}{2}\int_0^{2\pi}\int_0^a \left(\frac{I\rho}{2\pi a^2}\right)^2 \rho\, d\rho\, d\phi = \frac{\mu_0}{2}\left(\frac{I}{2\pi a^2}\right)^2 \cdot 2\pi \cdot \left(\frac{\rho^4}{4}\right)\Big|_0^a = \frac{\mu_0 I^2}{16\pi}$

For $a < \rho < b$, $U_{H2} = \frac{\mu_0}{2}\int_0^{2\pi}\int_a^b \left(\frac{I}{2\pi\rho}\right)^2 \rho\, d\rho\, d\phi = \frac{\mu_0}{2}\left(\frac{I}{2\pi}\right)^2 \cdot 2\pi \cdot (\ln\rho)\Big|_a^b = \frac{\mu_0 I^2}{4\pi}\ln(b/a)$

For $b < \rho < c$, $U_{H3} = \frac{\mu_0}{2}\int_0^{2\pi}\int_b^c \left(\frac{I}{2\pi\rho}\right)^2 \left(\frac{c^2-\rho^2}{c^2-b^2}\right)^2 \rho\, d\rho\, d\phi = \frac{\mu_0}{2}\left(\frac{I}{2\pi}\right)^2 \cdot 2\pi \cdot \int_b^c \frac{1}{\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)^2 d\rho$

$$= \frac{\mu_0 I^2}{4\pi} \cdot \frac{1}{(c^2-b^2)^2}\int_b^c \frac{1}{\rho}(c^2-\rho^2)^2 d\rho = \frac{\mu_0 I^2}{4\pi}\frac{1}{(c^2-b^2)^2}\int_b^c \left(\rho^3 - 2c^2\rho + \frac{c^4}{\rho}\right) d\rho$$

$$= \frac{\mu_0 I^2}{4\pi} \cdot \frac{1}{(c^2-b^2)^2}\left[\frac{1}{4}(c^4-b^4) - c^2(c^2-b^2) + c^4\ln(c/b)\right]$$

$$U_H = U_{H1} + U_{H2} + U_{H3} = \frac{\mu_0 I^2}{4\pi}\left\{\frac{1}{4} + \ln(b/a) + \frac{c^2+b^2}{4(c^2-b^2)} - \frac{c^2}{c^2-b^2} + \frac{c^4\ln(c/b)}{(c^2-b^2)^2}\right\} = \frac{1}{2}LI^2$$

$$\therefore L = \frac{\mu_0}{2\pi}\left\{\frac{1}{4} + \ln(b/a) + \frac{c^2+b^2}{4(c^2-b^2)} - \frac{c^2}{c^2-b^2} + \frac{c^4\ln(c/b)}{(c^2-b^2)^2}\right\}$$

13.26 From the result of Problem 13.18 (a), $U_H = \frac{\mu_0}{2}I^2\frac{a}{w} = \frac{1}{2}LI^2 \Rightarrow L = \mu_0\frac{a}{w}$

From $B^{(0)}$, we find the corresponding $H^{(0)}$ (Figure 14.17b):

$$H^{(0)} = 2400 \text{ A/m}$$

The iterative method calls for substituting the above value into (14.23) to obtain the "first-order" approximation of B, which is denoted as $B^{(1)}$:

$$B^{(1)} = \frac{(1000 - 2400 \times 0.121)\mu_0}{0.005} = 0.178 \text{ Wb/m}^2$$

The corresponding $H^{(1)}$ may be read from Figure 14.17b:

$$H^{(1)} = 2100 \text{ A/m}$$

We obtain the "second-order" approximation of B by substituting the above $H^{(1)}$ value for H in (14.23):

$$B^{(2)} = \frac{(1000 - 2100 \times 0.121)\mu_0}{0.005} = 0.187 \text{ Wb/m}^2$$

This procedure can be repeated to find the n-th iterative result of $B^{(n)}$. When a digital computer is available, the magnetization curve can be approximated by a standard polynomial-curve fitting and stored in the computer memory. A simple program may be written to carry out the iterative procedure, which requires very little computer time (see Problem 14.7).

The problem at hand can also be solved by a graphical method. Note that (14.22) or, equivalently, (14.23) is an equation of a straight line on the B-H plane. As shown in Figure 14.17b this line intersects the B axis at 0.251 Wb/m^2 and the H axis at 8264 A/m. It also intersects the nonlinear magnetization curve at $B = 0.19$ Wb/m^2. This result agrees fairly well with the result obtained by the iterative method.

Problems

14.1 Refer to the magnetization curve shown in Figure 14.3. The material is a nonlinear medium because μ depends on the magnitude of H. For magnetostatic fields, μ is equal to the slope of the line joining the origin to the (H, B) point on the magnetization curve. In this way, Figure 14.3b is obtained from Figure 14.3a. Now, if the material is placed in a time-harmonic field, the effective μ will be different from the μ for the magnetostatic fields. Consider a field $H = H_0 + H_1 \cos(\omega t + \phi)$, where H_0 is the bias magnetostatic field and H_1 is the amplitude of the time-harmonic component of the total field. Let $H_1 \ll H_0$; then the effective permeability of a material is the slope of the tangent of the magnetization curve at H_0. Sketch the effective μ versus H_0 for the curve shown in Figure 14.3a. Compare it with the magnetostatic μ shown in Figure 14.3b, and show that the μ's in these two cases are equal to each other at P_3.

14.2 Point out the differences between the following pairs of terms: (a) diamagnetic vs. paramagnetic, (b) remanence vs. retentivity, and (c) coercive force vs. coercivity.

14.3 What are approximate values of the retentivity and the coercivity of the ferrite shown in Figure 14.9?

14.4 Consider the carbon steel, alnico V, and cunico materials listed in Table 14.2. Which has the highest permanent magnetic-field strength? Which has the most difficulty in losing its permanent magnetism once it is magnetized?

14.5 A permanent magnet of radius 1.5 cm and thickness 0.3 cm is put in a magnetic field that is parallel to the disk, as in the situation depicted in Figure 14.7. The torque on the disk is equal to 1.2×10^{-3} N-m, and the magnetic field is equal to 10^{-3} Wb/m^2. What is the remanence of the permanent magnet?

14.6 To write "one" in the memory core x_2y_3 shown in Figure 14.11, how should the current pulses be sent along the wires? Specify the polarity of these pulses.

14.7 Consider the magnetic-core memory sketched in Figure 14.11 and the corresponding hysteresis curve for the cores shown in Figure 14.9. Now suppose that, because of malfunction in the circuitry, a positive pulse of amplitude I, which alone is capable of producing the switching magnetic field strength H_1, is sent down the line y_2 and that simultaneously an identical pulse is sent down the line x_3. Assume that all cores are initially in the "zero" state, which corresponds to having the magnetic flux circulation pointing either toward the upper left or the lower left (using the right-hand rule). What are the states of all of the cores after these pulses have passed through?

14.8 Compare the hysteresis loops of two ferrites shown in Figure P14.8. The curve labeled #1 is "thinner" than that labeled #2. Which ferrite core requires less switching current? Which ferrite has a better ability to withstand magnetic interferences?

14.9 Consider the magnetic circuit shown in Figure P14.9. The material is steel, and Figure 14.17 shows its magnetization curve. The flux density in the air gap is 0.5 Wb/m^2. Find the current I needed to produce this flux.

14.10 The magnetic circuit shown in Figure P14.10 is made of a material with $\mu = 600\mu_0$. Find the flux densities B_1 and B_2, and indicate their directions.

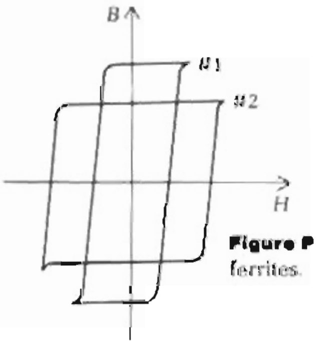


Figure P14.8 Hysteresis loops for two ferrites.

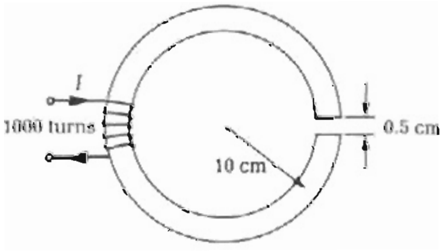
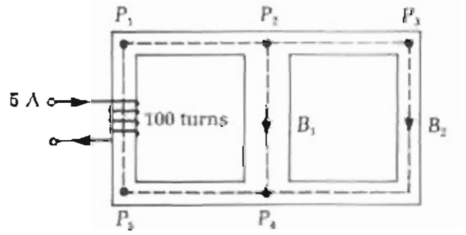


Figure P14.9



lengths:

$P_1P_2 = 9$ cm
$P_2P_4 = P_5P_1 = 10$ cm
$P_2P_3P_4 = 26$ cm
$P_4P_5 = 9$ cm

cross-sectional areas:

$P_2P_4 = 12$ cm^2
all other branches = 9 cm^2

Figure P14.10

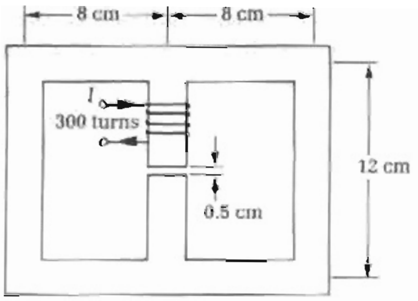


Figure P14.11

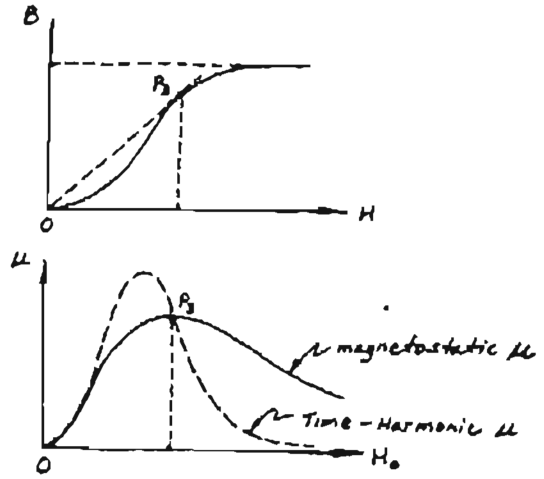
14.11 To produce a magnetic flux of 0.5 Wb/m^2 in the air gap of the magnetic circuit shown in Figure P14.11, what should be the magnitude of the current in the coil? Take $\mu = 200\mu_0$. The cross-sectional area of all branches is equal to 4 cm^2.

14.12 Write a computer program to carry out the iteration procedure outlined in Example 14.5. First, approximate the nonlinear curve in Figure 14.17b by a polynomial of fifth order. Then carry out the iteration five times to obtain the fourth-order approximation for B.

14.13 Find the approximate value of B in the magnetic circuit shown in Figure 14.17a for excitation current $I = 15$ A instead of 10 A. All other conditions given remain unchanged. Carry out the iteration a sufficient number of times to obtain an accuracy to the third digit.

CHAPTER 14

14.1



14.2 diamagnetic : magnetic fields produced by orbital and spinning electrons cancel out completely.

paramagnetic : magnetic fields due to the orbital and spinning electron do not cancel out completely.

remanence : When $H = 0$, the magnitude of the residual B is called remanence.

retentivity : the remanence B is called retentivity when both ends of the magnetization curve are at saturation.

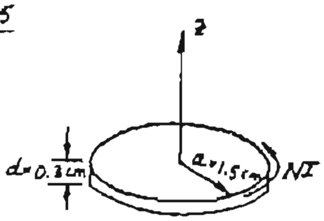
coercive force : The value of H in the negtive direction of initial magnetization to nullify the B field.

coercivity : the coercive force H is called coercivity when both ends of the magnetization curve are at saturation.

14.3 retentivity $= 0.22$ Wb/m^2 ; coercivity $= 100$ A/m

14.4 highest permanent magnetic field strength material is alnico V
most difficult to lose permanent magnetism material is cunico

14.5



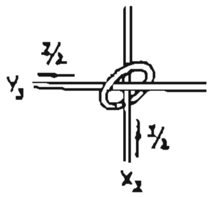
$$T = 1.2\times10^{-3} = mB = m\times10^{-3} \Rightarrow m = 1.2$$

$$m = NI(\pi a^2) \Rightarrow NI = \frac{m}{\pi a^2} = \frac{1.2}{\pi(0.015)^2} = 1697.65$$

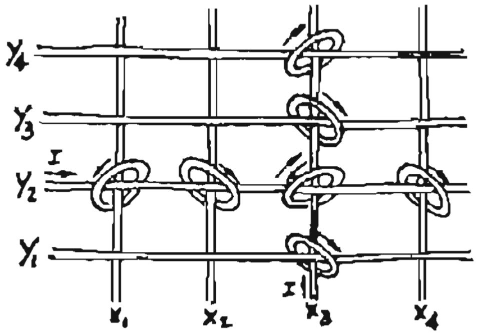
$$H = nI = NI/d = \frac{1697.65}{3\times10^{-3}} = 5.66\times10^5$$

$$\therefore\ B = \mu_0 H = 4\pi\times10^{-7}\times5.66\times10^5 = 0.71\ \text{Wb/m}^2$$

14.6



14.7



"0" state : magnetic moment pointing toward either upper-left or lower-left direction

$\therefore\ X_1Y_2 = 1$; $X_2Y_2 = 1$; $X_3Y_2 = 0$; $X_4Y_2 = 1$
$X_3Y_1 = 1$; $X_3Y_2 = 0$; $X_3Y_3 = 1$; $X_3Y_4 = 0$

14.8 #1 requires less switching current.

#2 has a better ability to withstand magnetic interferences.

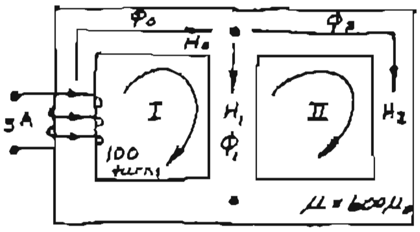
14.9 From Figure 14.17, with $B_a = 0.5\ Wb/m^2$, $H_s = 3600\ A/m$ in the steel. (B in air = B in steel)

From Figure P14.9, $1000I = \frac{B_a}{\mu_o}\times(5\times10^{-3}) + H_s\times(2\pi\times10^{-1} - 5\times10^{-3})$

$$= \frac{0.5\times5\times10^{-3}}{4\pi\times10^{-7}} + 3600\times(2\pi\times10^{-1} - 5\times10^{-3}) = 4233.4$$

$\therefore I = 4233.4/1000 = 4.23\ A$

14.10



Loop I : $5\times100 = H_o\times0.28 + H_1\times0.1$ .... ①

Loop II : $0 = H_2\times0.26 - H_1\times0.1 \Rightarrow H_1 = 2.6H_2$ ----- ②

$\phi_o = \phi_1 + \phi_2 \Rightarrow \mu H_o\times(9\times10^{-4}) = \mu H_1\times(12\times10^{-4}) + \mu H_2\times(9\times10^{-4})$

$\Rightarrow H_o = \frac{4}{3}H_1 + H_2 = (\frac{4}{3}\times2.6+1)H_2 = 4.467H_2$ --- ③

substituting ③ and ③ into ① yields

$500 = (0.28\times4.467 + 0.1\times2.6)H_2 \Rightarrow H_2 = \frac{500}{1.51} = 331$

$H_1 = 2.6H_2 = 2.6\times331 = 860.5$

$\therefore B_1 = \mu H_1 = 600\times4\pi\times10^{-7}\times860.5 = 0.6488\ Wb/m^2$ and $B_2 = \mu H_2 = 600\times4\pi\times10^{-7}\times331 = 0.2496\ Wb/m$

14.11 In the air gap, $B = B_a = 0.5\ Wb/m^2$ and $H_a = B_a/\mu_o = 0.5/\mu_o$

$\Phi_o = \Phi_1 + \Phi_2 \Rightarrow B_o\times(4\times10^{-4}) = (B_1+B_2)\times4\times10^{-4} \Rightarrow B_o = B_1 + B_2$

Because of the symmetry, $B_1 = B_2$ and $B_o = 2B_1$

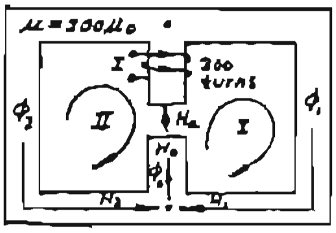
Since B in air = B in core $\Rightarrow B_o = B_a = 0.5\ Wb/m^2 \rightarrow H_o = 0.5/\mu$

$B_1 = \frac{B_o}{2} = \frac{B_a}{2} = 0.25\ Wb/m^2 \Rightarrow H_1 = 0.25/\mu$

From Loop I, $300I = H_a\times5\times10^{-3} + H_o\times(0.12 - 5\times10^{-3}) + H_1\times(0.08+0.12+0.08)$

$\therefore 300I = \frac{0.5}{\mu_o}\times5\times10^{-3} + \frac{0.5}{300\mu_o}\times0.115 + \frac{0.25}{300\mu_o}\times0.28 = \frac{1}{\mu_o}\times3.1375\times10^{-3}$

$\therefore I = 3.1375\times10^{-3}/(300\times4\pi\times10^{-7}) = 8.32\ A$



14.12

```
10      H=0
20      IMAGE 7A.DD,7A.D.DDD,17A.DDDDD.D,6A
30      FOR I=1  TO 5
40      J=I-1
60      B=(1000-.121*H)/3978.9
70      H=(25815.47-(129360.27-(354706.29-(447912.425-214034.26*B)*B)*B)*B)*B
80      PRINT USING 20;" ORDER=",J,"        B=",B," (Wb/m**2)        H=",H," (A/m)"
90      NEXT I
100     END
```

```
ORDER= 0        B= .251  (Wb/m**2)        H= 2375.6  (A/m)
ORDER= 1        B= .179  (Wb/m**2)        H= 2090.4  (A/m)
ORDER= 2        B= .188  (Wb/m**2)        H= 2127.8  (A/m)
ORDER= 3        B= .187  (Wb/m**2)        H= 2123.0  (A/m)
ORDER= 4        B= .187  (Wb/m**2)        H= 2123.6  (A/m)
```

14.13 $\frac{B}{\mu_o}\times0.005 + H_c\times0.121 = 15\times100 = 1500 \Rightarrow B = \frac{(1500 - 0.121H_c)}{0.005}\times\mu_o$

$B^{(0)} = 1500\times4\pi\times10^{-7}/0.005 = 0.377 \Rightarrow H^{(0)} \doteq 3000$ (From Figure 14.17)

$B^{(1)} = (1500 - 0.121\times3000)\times4\pi\times10^{-7}/0.005 = 0.286 \Rightarrow H^{(1)} \doteq 2600$ (From Figure 14.17)

$B^{(2)} = (1500 - 0.121\times2600)\times4\pi\times10^{-7}/0.005 = 0.298 \Rightarrow H^{(2)} \doteq 2600$ (From Figure 14.17)

Graphical method : $H = 0, B = 0.377$; $B = 0.2$, $H = (1500 - \frac{0.2}{\mu_o}\times0.005)/0.121 = 5820$

$\Rightarrow B \doteq 0.3\ Wb/m^2$

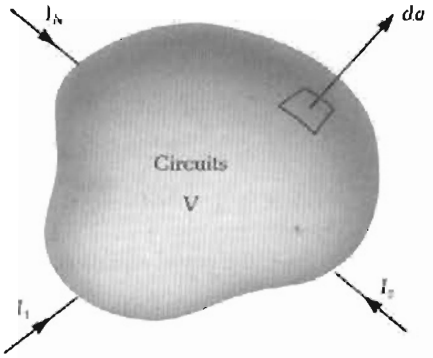


Figure 15.9 Poynting's theorem for quasistatic fields.

$$
\begin{aligned}
P &= -\oiint_A d\boldsymbol{a} \cdot (\mathbf{E} \times \boldsymbol{H}) = \oiint_A d\boldsymbol{a} \cdot [(\nabla\Phi) \times \boldsymbol{H}] \\
&= \iiint_V dV\, \nabla \cdot [(\nabla\Phi) \times \boldsymbol{H}] = \iiint_V dV\, \nabla \cdot [\nabla \times (\Phi \boldsymbol{H}) - \Phi\nabla \times \boldsymbol{H}] \\
&= -\iiint_V dV\, \nabla \cdot (\Phi\nabla \times \boldsymbol{H}) = -\oiint_A d\boldsymbol{a} \cdot \Phi\nabla \times \boldsymbol{H} \\
&= -\oiint d\boldsymbol{a} \cdot \Phi\left(\boldsymbol{J} + \frac{\partial \mathbf{D}}{\partial t}\right)
\end{aligned}
$$

Ignoring the displacement current $\partial \mathbf{D}/\partial t$, we find

$$
P = -\oiint d\boldsymbol{a} \cdot \Phi \boldsymbol{J} = \sum_{n=1}^{N} V_n\left(-\iint_{A_n} d\boldsymbol{a} \cdot \boldsymbol{J}\right) = \sum_{n=1}^{N} V_n I_n
$$

where I_n represents the current flowing into the volume through the surface A_n, whose surface normal is pointing outward. Thus, we have established that the circuit theory's concept of power input is valid only when the displacement current is negligible.

Problems

15.1 Consider the short-circuited parallel plate shown in Figure P15.1, with width w, length ℓ, and separation a. Find the zeroth-, first-, second-, and third-order electric and magnetic fields. Show that the sum of the quasistatic solution is equal to the full wave solution as presented in Chapter 6 for a short-circuit transmission line. Assume that the current at $z = 0$ is $I(t) = I_0 \cos(\omega t)$ and that all fields are functions of t and z only.

15.2 Calculate the total zeroth-order stored electric energy in the parallel-plate region shown in Figure 15.1. What is the zeroth-order stored magnetic energy in the same region?

15.3 Calculate the total first-order stored magnetic energy in the parallel-plate region shown in Figure 15.1. What is the first-order stored electric energy in the same volume? Denote $U_{Hm}^{[1]}$ the maximum total first-order stored magnetic energy in the

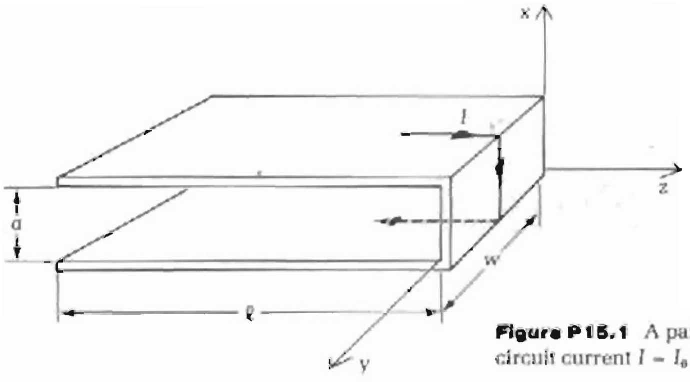


Figure P15.1 A parallel plate with short-circuit current $I = I_0 \cos(\omega t)$ at $z = 0$.

region and $U_{Em}^{(0)}$ the maximum total zeroth-order stored electric energy in the same volume. Show that

$$\frac{U_{Hm}^{(1)}}{U_{Em}^{(0)}} = \tfrac{1}{3}(k\ell)^2$$

15.4 Calculate the total second-order stored electric energy in the parallel-plate region shown in Figure 15.1. Compare it with the zeroth-order stored electric energy. If ℓ is 0.1λ long, what can you say about the relative magnitudes of the zeroth-order stored electric energy, the first-order stored magnetic energy, and the second-order stored electric energy?

15.5 Find the total zeroth-order stored electric and magnetic energies in a parallel plate with a short-circuit current $I = I_0 \cos(\omega t)$ at $z = 0$ (refer to Problem 15.1).

15.6 Find the higher-order stored electric and magnetic energies in the parallel-plate region shown in Figure P15.1 up to the third order. If $\ell = 0.1\lambda$, compare the relative magnitudes of these stored energies. Use the total zeroth-order stored magnetic energy found in Problem 15.5 for comparison.

15.7 A coaxial line ℓ meters long is filled with a material characterized by ϵ and σ. The radii of the inner and the outer conductors are a and b, respectively. The voltage between the coaxial conductors is $V_0 \cos(\omega t)$. Find the zeroth-order electroquasistatic field $E^{(0)}$, the current $I^{(0)}$, the charge $Q^{(0)}$, and the first-order current $I^{(1)}$. Express these in terms of the parameters V_0, a, b, ℓ, ϵ, σ, and ωt.

15.8 Two concentric spherical electrodes of radii a and b, respectively, are filled with a material characterized by ϵ and σ. The voltage between the electrodes is $V_0 \cos(\omega t)$. Find the zeroth-order electroquasistatic field $E^{(0)}$, the current $I^{(0)}$, the charge $Q^{(0)}$, and the first-order current $I^{(1)}$. Express these in terms of V_0, a, b, ϵ, σ, and ωt.

15.9 Show that the time needed to charge a Van de Graaff generator shown in Figure 9.24a with radius R to a maximum voltage of V_{max} by applying a charging current I is equal to $4\pi\epsilon_0 R V_{max}/I$. Calculate the charging time t if $R = 1$ m, $V_{max} = 10^6$ V and $I = 10^{-5}$ A

CHAPTER 15

15.1 $I(t) = I_o \cos\omega t \Rightarrow \bar{J}(t) = \hat{z}\frac{I_o}{W}\cos\omega t$

zeroth order: $\bar{E}^{(0)} = 0$

from the boundary condition, on the lower plate, $\hat{z}\times\bar{H}^{(0)} = \bar{J}^{(0)}$

$\therefore \bar{H}^{(0)} = -\hat{y}\frac{I_o}{W}\cos\omega t$

1st order: $\nabla\times\bar{E}^{(1)} = -\frac{\partial}{\partial t}\mu\bar{H}^{(0)} \Rightarrow \hat{y}\frac{\partial E_x^{(1)}}{\partial z} = -\hat{y}\frac{I_o}{W}\mu\omega\sin\omega t \Rightarrow \bar{E}^{(1)} = -\hat{x}\frac{I_o}{W}\mu\omega z\sin\omega t$

$\bar{H}^{(1)} = 0$

2nd order: $\bar{E}^{(2)} = 0$

$\nabla\times\bar{H}^{(2)} = \frac{\partial}{\partial t}\varepsilon\bar{E}^{(1)} \Rightarrow \hat{x}\left(-\frac{\partial H_y^{(2)}}{\partial z}\right) = -\hat{x}\frac{I_o}{W}\mu\varepsilon\omega^2 z\cos\omega t \Rightarrow \bar{H}^{(2)} = \hat{y}\frac{I_o}{W}\mu\varepsilon\omega^2\frac{z^2}{2}\cos\omega t$

3rd order: $\nabla\times\bar{E}^{(3)} = -\frac{\partial}{\partial t}\mu\bar{H}^{(2)} \Rightarrow \hat{y}\left(\frac{\partial E_x^{(3)}}{\partial z}\right) = \hat{y}\frac{I_o}{W}\mu^2\varepsilon\omega^3\frac{z^2}{2}\sin\omega t \Rightarrow \bar{E}^{(3)} = \hat{x}\frac{I_o}{W}\mu^2\varepsilon\omega^3\frac{z^3}{2\cdot 3}\sin\omega t$

$\bar{H}^{(3)} = 0$

$\therefore \bar{E} = -\hat{x}\frac{I_o}{W}\omega\mu\sin\omega t\left(z - \frac{z^3}{3!}\omega^2\mu\varepsilon + \cdots\right) = -\hat{x}\frac{I_o}{W}\frac{\omega\mu}{k}\sin\omega t\left[kz - \frac{1}{3!}(kz)^3 + \cdots\right] = -\hat{x}\frac{I_o}{W}\eta\sin\omega t\sin(kz)$

$\bar{H} = -\hat{y}\frac{I_o}{W}\cos\omega t\left(1 - \frac{z^2}{2}\omega^2\mu\varepsilon + \cdots\right) = -\hat{y}\frac{I_o}{W}\cos\omega t\cos(kz)$

In phasor notations, $\underline{E} = -\hat{x}\frac{I_o}{2W}\eta(e^{-jkz} - e^{jkz})$ and $\underline{H} = -\hat{y}\frac{I_o}{2W}(e^{-jkz} + e^{jkz})$; these are the full wave solution to a short-circuit transmission line.

15.2 $\bar{E}^{(0)} = -\hat{x}\frac{V_o}{a}\cos\omega t$, $\bar{H}^{(0)} = 0$

$\therefore U_E^{(0)} = \frac{1}{2}\int\varepsilon\bar{E}^{(0)}\cdot\bar{E}^{(0)}dv = \frac{1}{2}\varepsilon\left(\frac{V_o}{a}\right)^2\cos^2\omega t\,(aw\ell) = \frac{\varepsilon}{2}\left(\frac{w\ell}{a}\right)V_o^2\cos^2\omega t$ and $U_H^{(0)} = 0$

15.3 $\bar{E}^{(1)} = 0$, $\bar{H}^{(1)} = -\hat{y}\,\omega z\frac{\varepsilon V_o}{a}\sin\omega t$

$U_E^{(1)} = 0$ and $U_H^{(1)} = \frac{1}{2}\int\mu\bar{H}^{(1)}\cdot\bar{H}^{(1)}dv = \frac{\mu}{2}\left(\frac{\varepsilon V_o}{a}\right)^2\omega^2\sin^2\omega t\cdot(aw)\cdot\int_0^\ell z^2dz = \frac{\omega^2\mu\varepsilon^2}{6}\left(\frac{w\ell}{a}\right)\ell^2V_o^2\sin^2\omega t$

From the result of problem 15.2, $U_{Em}^{(0)} = \frac{\varepsilon}{2}\left(\frac{w\ell}{a}\right)V_o^2$.

$U_{Hm}^{(1)} = \frac{\omega^2\mu\varepsilon^2}{6}\left(\frac{w\ell}{a}\right)\ell^2V_o^2 \Rightarrow U_{Hm}^{(1)}/U_{Em}^{(0)} = \left[\frac{\omega^2\mu\varepsilon^2}{6}\left(\frac{w\ell}{a}\right)\ell^2V_o^2\right]\Big/\left[\frac{\varepsilon}{2}\left(\frac{w\ell}{a}\right)V_o^2\right] = \frac{1}{3}\omega^2\mu\varepsilon\ell^2 = \frac{1}{3}(k\ell)^2$

15.4 $\bar{E}^{(2)} = \hat{x}\left(\frac{V_o}{2a}\right)k^2z^2\cos\omega t \Rightarrow U_E^{(2)} = \frac{1}{2}\int\varepsilon\bar{E}^{(2)}\cdot\bar{E}^{(2)}dv = \frac{\varepsilon}{2}\left(\frac{V_o}{2a}\right)^2k^4\cos^2\omega t\,(aw)\int_0^\ell z^4dz = \frac{\varepsilon}{40}\left(\frac{w\ell}{a}\right)\ell^4k^4V_o^2\cos^2\omega t$

$U_E^{(2)}/U_E^{(0)} = \left[\frac{\varepsilon}{40}\left(\frac{w\ell}{a}\right)\ell^4k^4V_o^2\cos^2\omega t\right]\Big/\left[\frac{\varepsilon}{2}\left(\frac{w\ell}{a}\right)V_o^2\cos^2\omega t\right] = \frac{1}{20}(k\ell)^4 = U_{Em}^{(2)}/U_{Em}^{(0)}$

if $\ell = 0.1\lambda$, $k\ell = \frac{2\pi}{\lambda}\times 0.1\lambda = 0.2\pi$

$\therefore U_{Hm}^{(1)}/U_{Em}^{(0)} = \frac{1}{3}(k\ell)^2 = \frac{1}{3}(0.2\pi)^2 = 0.13$

$U_{Em}^{(2)}/U_{Em}^{(0)} = \frac{1}{20}(k\ell)^4 = \frac{1}{20}(0.2\pi)^4 = 0.0078$

15.5 $U_E^{(0)} = 0$, $U_H^{(0)} = \frac{1}{2}\int\mu\bar{H}^{(0)}\cdot\bar{H}^{(0)}dv = \frac{\mu}{2}\left(\frac{a\ell}{w}\right)I_o^2\cos^2\omega t$

15.6 $U_H^{(1)} = 0$, $U_E^{(1)} = \frac{1}{2}\int\varepsilon\bar{E}^{(1)}\cdot\bar{E}^{(1)}dv = \frac{\omega^2\mu^2\varepsilon}{2}\frac{aw}{w^2}I_o^2\sin^2\omega t\int_0^\ell z^2dz = \frac{\omega^2\mu^2\varepsilon}{6}\left(\frac{a\ell}{w}\right)\ell^2I_o^2\sin^2\omega t$

$U_E^{(2)} = 0$, $U_H^{(2)} = \frac{1}{2}\int\mu\bar{H}^{(2)}\cdot\bar{H}^{(2)}dv = \frac{\omega^4\mu^3\varepsilon^2}{8}\frac{a}{w}I_o^2\cos^2\omega t\int_0^\ell z^4dz = \frac{\omega^4\mu^3\varepsilon^2}{40}\left(\frac{a\ell}{w}\right)\ell^4I_o^2\cos^2\omega t$

$U_H^{(3)} = 0$, $U_E^{(3)} = \frac{1}{2}\int\varepsilon\bar{E}^{(3)}\cdot\bar{E}^{(3)}dv = \frac{\omega^6\mu^4\varepsilon^3}{72}\frac{a}{w}I_o^2\sin^2\omega t\int_0^\ell z^6dz = \frac{\omega^6\mu^4\varepsilon^3}{504}\left(\frac{a\ell}{w}\right)\ell^6I_o^2\sin^2\omega t$

if $\ell = 0.1\lambda$, $k\ell = \frac{2\pi}{\lambda}\times 0.1\lambda = 0.2\pi$

$U_{Em}^{(1)}/U_{Hm}^{(0)} = \frac{1}{3}(k\ell)^2 = \frac{1}{3}(0.2\pi)^2 = 0.13$

$U_{Hm}^{(2)}/U_{Hm}^{(0)} = \frac{1}{20}(k\ell)^4 = \frac{1}{20}(0.2\pi)^4 = 0.0078$

$U_{Em}^{(3)}/U_{Hm}^{(0)} = \frac{1}{252}(k\ell)^6 = \frac{1}{252}(0.2\pi)^6 = 0.000096$

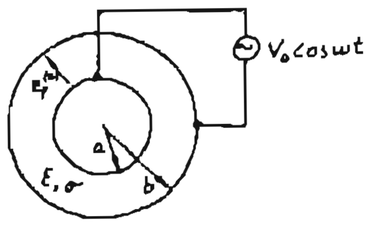
15.7 $E_\rho^{(0)} = A/\rho \Rightarrow V_o\cos\omega t = -\int_b^a\frac{A}{\rho}d\rho = A\ln(b/a) \Rightarrow A = \frac{V_o\cos\omega t}{\ln(b/a)}$

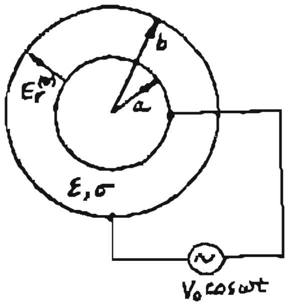
$\therefore E_\rho^{(0)} = \frac{V_o\cos\omega t}{\rho\ln(b/a)} \Rightarrow J_\rho^{(0)} = \sigma E_\rho^{(0)} = \frac{\sigma V_o\cos\omega t}{\rho\ln(b/a)}$

$I^{(0)} = 2\pi a\ell\cdot J_\rho^{(0)}|_{\rho=a} = 2\pi a\ell\frac{\sigma V_o\cos\omega t}{a\ln(b/a)} = 2\pi\ell\sigma V_o\cos\omega t/\ln(b/a)$

$Q^{(0)} = 2\pi a\ell\cdot\varepsilon E_\rho^{(0)}|_{\rho=a} = 2\pi a\ell\frac{\varepsilon V_o\cos\omega t}{a\ln(b/a)} = 2\pi\ell\varepsilon V_o\cos\omega t/\ln(b/a)$

$I^{(1)} = \frac{\partial Q^{(0)}}{\partial t} = -2\pi\omega\ell\varepsilon V_o\sin\omega t/\ln(b/a)$





15.8 $E_r^{(0)} = \dfrac{Q^{(0)}}{4\pi\varepsilon r^2}$, $a<r<b$

$V_0 \cos\omega t = -\int_b^a E_r^{(0)} dr = \dfrac{Q^{(0)}}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right)$

$\therefore Q^{(0)} = 4\pi\varepsilon V_0 \cos\omega t / \left(\frac{1}{a}-\frac{1}{b}\right)$

$E^{(0)} = \dfrac{Q^{(0)}}{4\pi\varepsilon r^2} = \dfrac{V_0 \cos\omega t}{(1/a - 1/b) r^2}$

$J^{(0)} = \sigma E^{(0)} = \sigma V_0 \cos\omega t / [(1/a - 1/b) r^2]$

$\therefore I^{(0)} = 4\pi a^2 \cdot J^{(0)}\big|_{r=a} = 4\pi a^2 \dfrac{\sigma V_0 \cos\omega t}{(1/a - 1/b) a^2} = 4\pi\sigma V_0 \cos\omega t / \left(\frac{1}{a}-\frac{1}{b}\right)$

$I^{(1)} = \dfrac{\partial Q^{(0)}}{\partial t} = -4\pi\omega\varepsilon V_0 \sin\omega t / \left(\frac{1}{a}-\frac{1}{b}\right)$

15.9 $V = \dfrac{Q}{4\pi\varepsilon_0 R} \Rightarrow \dfrac{dV}{dt} = \dfrac{1}{4\pi\varepsilon_0 R}\dfrac{dQ}{dt} = \dfrac{I}{4\pi\varepsilon_0 R}$.

$\therefore \int_0^{V_{max}} dV = V_{max} = \int_0^t \dfrac{I}{4\pi\varepsilon_0 R} dt = \dfrac{I}{4\pi\varepsilon_0 R} t \Rightarrow t = 4\pi\varepsilon_0 R V_{max} / I$

For $R = 1$ m, $V_{max} = 10^6$ V and $I = 10^{-5}$ A,

$t = 4\pi \times \frac{1}{36\pi} \times 10^{-9} \times 1 \times 10^6 / 10^{-5} = \frac{100}{9} = 11.1$ sec

Solution: From (16.72), we find

$$F_{\text{coil}} = \frac{2 \times 4\pi \times 10^{-7} \times 2 \times 10^5 \times 2 \times 10^5 \times 1.0}{\pi \times 0.5} = 64 \times 10^3 \text{ N}$$

$$= 6.5 \text{ metric tons}$$

Problems

16.1 A small circular loop of 5 mm radius is placed 1 m away from a 60-Hz power line. The voltage induced on this loop is measured at 0.6 microvolt. What is the current on the power line?

16.2 Assume that the current on the infinitely long line shown in Figure 16.1 is the triangular pulse shown in Figure P16.2. Find the induced voltage on the rectangular loop. Use the following data: $a = 2$ cm, $b = 4$ cm, and $d = 1$ cm.

16.3 Consider the network shown in Figure P16.3. The magnetic flux is increasing at a rate of 0.5 Wb/s in the direction pointing into the paper. Find the readings of the voltmeters shown.

16.4 Find the readings of the voltmeters shown in Figure P16.4. The magnetic flux is increasing at a rate of 0.5 Wb/s in the direction pointing into the paper.

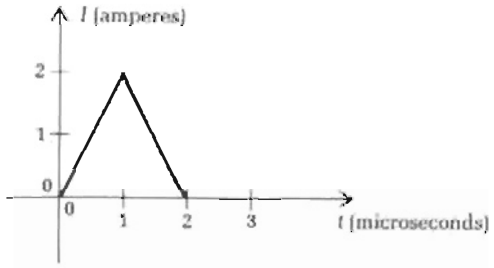


Figure P16.2

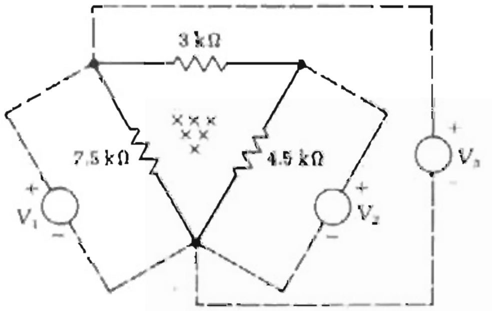


Figure P16.3

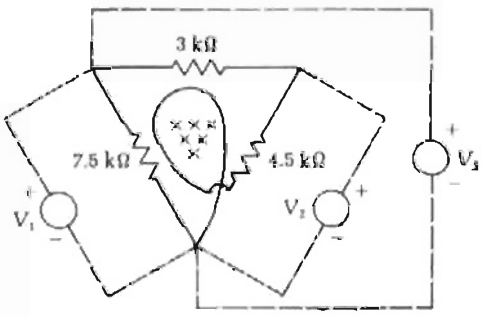


Figure P16.4

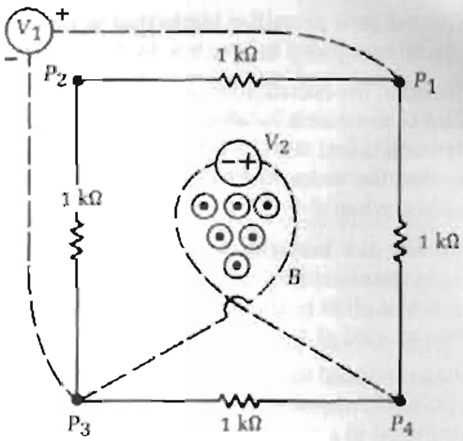


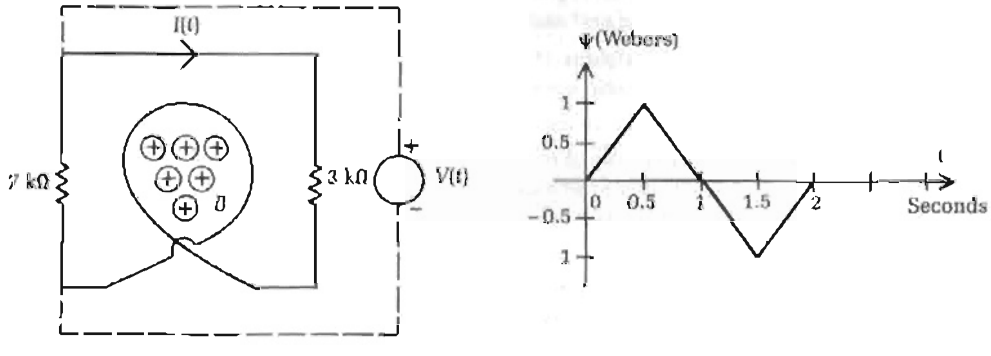
Figure P16.5

16.5 Four resistors form a circuit as shown in Figure P16.5. The total magnetic flux linking the circuit is increasing at a rate of 0.5 Wb/s, in the direction pointing out of the paper.

(a) Find the direction and magnitude of the induced current in the circuit.
(b) Find the readings of the voltmeters V_1 and V_2.

16.6 Two resistors are connected by wires to form a circuit as shown in Figure P16.6a. The magnetic flux linking the circuit varies with time. Figure P16.6b shows the time variation of the magnetic flux. The positive value of the flux corresponds to the flux directed into the paper. The magnitude of the flux is for a **single** turn of a circuit loop that encircles the magnetic flux.

(a) Plot the current $I(t)$ versus time. Be sure to mark the scale of the current.
(b) Plot the voltage $V(t)$ versus time. Mark the scale.



(a) (b)

Figure P16.6

16.7 What is the EMF induced on a propeller blade that is 1.5 m long and is rotating at 10,000 r/min in the earth's magnetic field (0.5×10^{-4} Wb/m^2)?

16.8 Find the voltage induced in the rectangular loop shown in Figure 16.1 if it is rotating about the axis parallel to the z axis located at $x = d + \frac{a}{2}$. Assume that the angular frequency of the rotation is ω and that the infinitely long wire carries a direct current of I amperes. Show that the induced EMF is not a pure sinusoidal voltage. It is approximately sinusoidal when $d \gg a$.

16.9 A magnetic core is made of a material whose hysteresis loop is shown in Figure P16.9. Note that this hysteresis curve is not a "square loop." To read the content of the core, two pulses are applied to the wires. The currents generate an H equal to 200 A/m. The core has an area of 3×10^{-7} m^2.

(a) What is the voltage induced in the sensing wire if the core is originally at the "zero" state (at point C)? Assume that switching from C to A is linear with time and that it is completed in a microsecond.

(b) What is the voltage induced in the sensing wire if the core is originally at the "one" state (at point A)? Assume that switching from A to A' is linear with time and that it is completed in 0.5 μs. This voltage is the "noise" voltage because it would ideally be zero if the hysteresis loop were a perfect square.

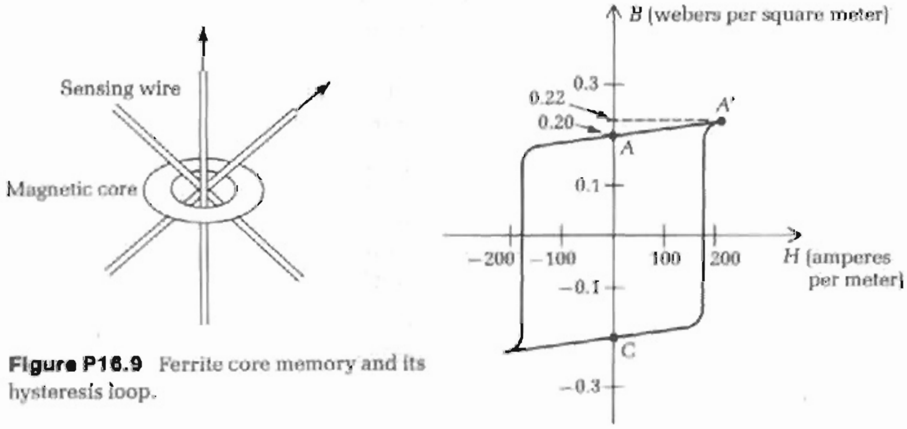


Figure P16.9 Ferrite core memory and its hysteresis loop.

16.10 Find the total expansion force acting on the surface of an air-core solenoid that has 100 turns of coil and radius $a = 1$ cm, length $\ell = 10$ cm, and current $I = 10$ A.

16.11 Repeat Problem 16.10 for the case in which 100 turns of coil are wound over a ferromagnetic core with $\mu = 1000\mu_0$. The current is 10 mA, with $a = 1$ cm and $\ell = 10$ cm.

16.12 A copper pipe of radius $a = 2$ cm and thickness $d = 0.1$ cm is placed in a solenoid that has 200 turns per meter and is excited by a 1000 Hz 10-A current. The conductivity of the copper is 5.92×10^7 mho/m. Calculate the power per meter dissipated in the copper pipe.

16.13 A transformer similar to the one shown in Figure 16.11 is made of a steel with relative permeability equal to 1100. The effective length of the core is 40 cm, and the flux density is $B = 0.3$ Wb/m^2. $N_1 = 100$, $N_2 = 1{,}000$, $I_1 = 60$ A.

(a) Find I_2, assuming that the transformer is an ideal transformer.

(b) Find I_2, using (16.37).

(c) Compare the two answers.

16.14 Consider a magnetic circuit similar to the one shown in Figure 16.11. The effective length of the core is 0.4 m and its permeability is 2000 μ_0. The cross-sectional area of the core is 4×10^{-4} m². Let $I_1 = 10$ A, $I_2 = 24$ A, $N_1 = 50$, and $N_2 = 20$.

(a) Calculate the B field in the core. Give both the direction and the magnitude.
(b) If I_1 is a-c with f = 60 Hz, what are $|V_1|$ and $|V_2|$? Assume that the magnetic flux always stays in the core without any leakage.

16.15 The primary coil of a transformer has 150 turns and the secondary coil has 450 turns. The effective length of the core is 0.5 m and the flux density in the core is 0.25 Wb/m². The transformer is similar to the one shown in Figure 16.11. Assume that $I_1 = 60$ A and there is no flux leakage.

(a) Find I_2, assuming ideal transformer condition.
(b) Find V_2, assuming ideal transformer condition and $V_1 = \cos(120\omega t)$.
(c) Find I_2, taking into consideration that the core material has a finite permeability equal to 1000 μ_0.
(d) The hysteresis loop of the core material has an area equal to 90 Wb-A/m³. What is the power loss due to the hysteresis in the transformer? Assume that the core has a cross-sectional area equal to 4 cm².

16.16 Figure P16.16 shows a magnetization curve of a core used in a transformer. Notice that the hysteresis is negligible in this case and that the curve is linear in the range $0 \le |H| \le 150$ A/m but saturated when H is increased beyond this range. Let us now review Example 16.10. Because $|V_1| = \omega N_1 \Psi$, we want to use maximum Ψ in order to minimize the number of coils in the transformer. Using Figure P16.16, explain what will happen to the shape of the $\Psi(t)$ and consequently to the shape of $V_1(t)$ if Ψ is too high—for example, if Ψ is so high as to correspond to $B = 1.2$ Wb/m².

16.17 Estimate the approximate power loss attributed to hysteresis in the ferrite core shown in Figure P16.9 if the core is switched back and forth between "zero" and "one" states 1000 times in a second. Assume that the core has an average radius of 6×10^{-4} m and that its cross-sectional area is 3×10^{-7} m².

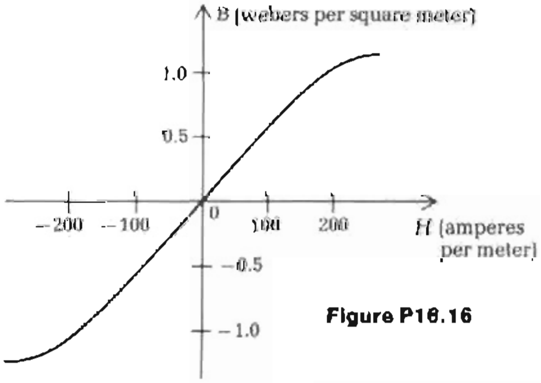


Figure P16.16

16.18 Show that the mechanical torque required to drive an ac generator is not constant with time or, to be exact, that it consists of a constant term and a term that varies sinusoidally with time with an angular frequency 2ω. What is the time average of the torque? Express the torque in terms of the area of the winding A, the current I, the magnetic flux density B, and the phase angle α between the voltage and the current. Plot T as a function of t for $\alpha = 0$.

16.19 Figure 16.15 depicts an ac generator with a single coil being rotated in a constant magnetic field. It illustrates the operating principle of a single-phase ac generator. Let us now consider a three-phase ac generator. How would you physically arrange three sets of coils in order to generate three-phase electricity? To illustrate your design, sketch a diagram similar to Figure 16.15.

16.20 What is the total mechanical torque needed to drive the three-phase generator that you have designed for Problem 16.19? Express this torque as a function of time in terms of the appropriate parameters. Plot T as a function of time, and compare it with that obtained in Problem 16.18. Is the instantaneous mechanical torque "smoother" (does its time-average value fluctuate less) compared with that for a single-phase generator?

16.21 Design a coil configuration similar to the one shown in Figure 16.17. Design it in such a way that it will produce a rotating magnetic field in the armature-stator air gap and that the field will have an angular speed equal to $\omega/2$ when fed with the 3-phase current shown in Figure 16.18. Prove that your design is correct by drawing instantaneous-current diagrams similar to those shown in Figure 16.17.

16.22 Show qualitatively that the torque generated by an induction motor may be varied by changing the resistance of the windings of the rotor. Figure 16.21 shows torque curves versus v_r/v_s with three different values of rotor conductivities. What are the relative magnitudes of σ_1, σ_2, and σ_3?

16.23 Refer to the synchronous motor shown in Figure 16.22. What happens when the torque angle is negative—that is, when the position of the rotor magnetic-moment is ahead of the magnetic field?

16.24 Consider the coils of the magneplane's track shown in Figure 16.25. For the magneplane to travel at a speed of 250 km/h, what should be the distance ℓ_t in meters? Assume that the power is provided by a three-phase 60-Hz power line.

CHAPTER 16

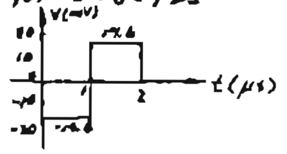
16.1 $B = \mu_0 I/(2\pi\rho) \Rightarrow \psi = BA = \frac{\mu_0 I A}{2\pi\rho} \Rightarrow \underline{V} = -\frac{\partial\psi}{\partial t} = -j\omega\psi = \frac{-j\omega\mu_0 \underline{I} A}{2\pi\rho} \Rightarrow \underline{I} = j\frac{2\pi\rho\underline{V}}{\omega\mu_0 A}$

$\therefore\ I = |\underline{I}| = \frac{2\pi\rho|\underline{V}|}{\omega\mu_0 A} = \frac{2\pi\times1\times(0.6\times10^{-6})}{2\pi\times60\times4\pi\times10^{-7}\times\pi\times(5\times10^{-3})^2} = 101.3\,A$

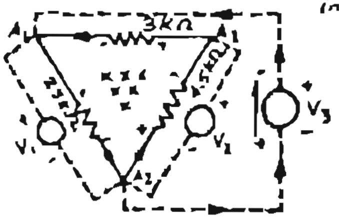
16.2 $\psi = \frac{\mu b}{2\pi} I(t) \ln\left(\frac{d+a}{d}\right)$ with $a = 2$ cm, $b = 4$ cm, and $d = 1$ cm

From Figure P16.2, $I(t) = \frac{2t}{10^{-6}}$ for $0 < t < 1\,\mu s$; $I(t) = \frac{-2}{10^{-6}}(t - 2\times10^{-6})$ for $1\,\mu s < t < 2\,\mu s$

$$\therefore V = \frac{-\partial\psi}{\partial t} = \begin{cases} -\frac{\mu b}{2\pi}\cdot\frac{2}{10^{-6}}\ln\left(\frac{d+a}{d}\right) = -\frac{4\pi\times10^{-7}\times4\times10^{-2}\times2}{2\pi\times10^{-6}}\ln(3/1) = -17.6\,mV & \text{for } 0<t<1\,\mu s \\ \frac{\mu b}{2\pi}\cdot\frac{2}{10^{-6}}\ln\left(\frac{d+a}{d}\right) = 17.6\,mV & \text{for } 1\,\mu s < t < 2\,\mu s \end{cases}$$



16.3



induced voltage along loop $A_1A_2A_3$ is $V = 0.5$ V

induced current in the loop $A_1A_2A_3$ is

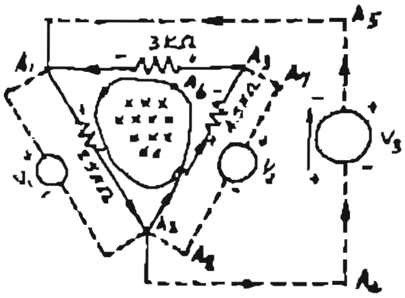
$I = 0.5/(3+4.5+7.5) = \frac{1}{30}$ mA in the counterclockwise direction.

$\therefore\ V_1 = I\times7.5\,k\Omega = 0.25$ V

$V_2 = -I\times4.5\,k\Omega = -0.15$ V

also, $V = V_1 - V_3 \Rightarrow V_3 = V_1 - V = -0.25$ V

16.4



induced voltage along loop $A_1A_2A_3$ is $V' = 1.0$ V

induced current in the loop $A_1A_2A_3$ is $I = \frac{1}{3+4.5+7.5} = \frac{1}{15}$ mA

$V_1 - I\times7.5\,k\Omega = 0 \Rightarrow V_1 = \frac{1}{15}\times7.5 = 0.5$ V

induced voltage along the loop $A_2A_6A_3A_7A_8$ is $V'' = 0.5$ V

$\therefore\ V'' - I\times4.5\,k\Omega - V_2 = 0 \Rightarrow V_2 = 0.5 - \frac{1}{15}\times4.5 = 0.2$ V

Induced voltage along loop $A_1A_2A_4A_5$ is $V''' = 0.5$ V

$\therefore\ V''' - I\times7.5\,k\Omega = V_3 \Rightarrow V_3 = 0.5 - \frac{1}{15}\times7.5 = 0$ V

16.5 (a) $EMF = -\frac{\Delta\Psi}{\Delta t} = -0.5$

$I = \frac{0.5}{4\times10^3} = 0.125$ mA (clockwise)

(b) loop $P_1V_1P_3P_2P_1$: $0 = V_1 + 0.125 + 0.125 \rightarrow V_1 = -0.25$ V

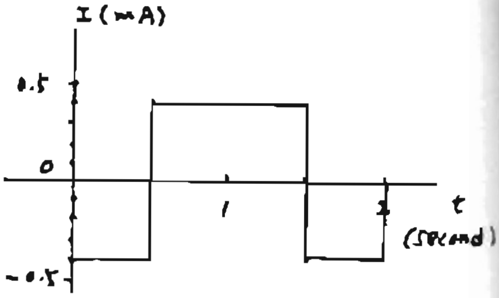
loop $P_1P_2P_3V_2P_4P_1$: $-0.5\times2 = -0.125 - 0.125 + V_2 - 0.125 \rightarrow V_2 = -0.625$ V

or loop $V_2P_4P_3V_2$: $-0.5 = V_2 + 0.125 \rightarrow V_2 = -0.625$ V

16.6 (a) $EMF = -2\frac{\Delta\Psi}{dt}$; $\frac{\Delta\Psi}{dt} = \begin{cases} 2 & 0<t<0.5 \\ -2 & 0.5<t<1.5 \\ 2 & 1.5<t<2 \end{cases}$

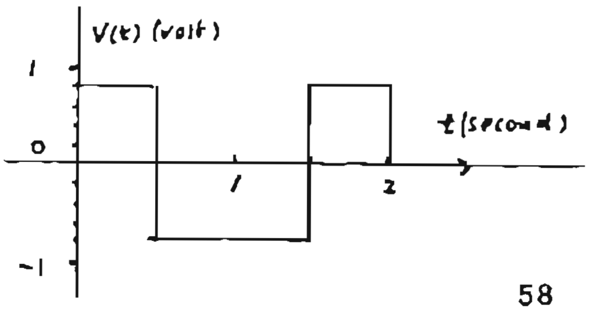
$I(t) = EMF/10\,k\Omega$

$I(t) = \begin{cases} -0.4\text{ mA} & 0<t<0.5 \\ +0.4\text{ mA} & 0.5<t<1.5 \\ -0.4\text{ mA} & 1.5<t<2 \end{cases}$



(b) $EMF' = $ EMF of V-meter and $7\,k\Omega$ loop $= \frac{1}{2}EMF = -\frac{\Delta\Psi}{dt}$

$EMF' = V(t) + 7\times10^3 I(t) \rightarrow V(t) = \begin{cases} 0.8\text{ V} & 0<t<0.5 \\ -0.8\text{ V} & 0.5<t<1.5 \\ 0.8\text{ V} & 1.5<t<2 \end{cases}$

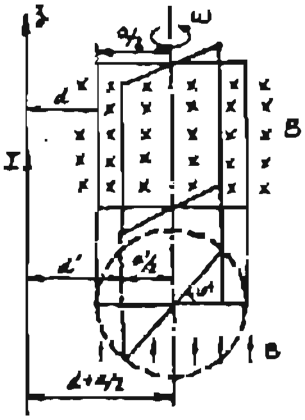


16.7 $v = r\omega$; $d\mathcal{E} = vB\,dr = \omega Br\,dr \Rightarrow \mathcal{E} = \frac{1}{2}\omega Br^2 = \frac{1}{2}\times 2\pi\times 10^4 \times \frac{1}{60}\times 0.5\times 10^{-4}\times(1.5)^2 = 58.9\ \text{mV}$

16.8 At time t, the rotation angle is ωt, and

$$d' = d + \frac{a}{2} - \frac{a}{2}\cos\omega t$$

$$d'+a' = d + \frac{a}{2} + \frac{a}{2}\cos\omega t$$



∴ Total flux linkage through the loop at time t is

$$\psi = \frac{\mu I}{2\pi} b \ln\left(\frac{d'+a'}{d'}\right) = \frac{\mu I}{2\pi} b \ln\left(\frac{d+\frac{a}{2}+\frac{a}{2}\cos\omega t}{d+\frac{a}{2}-\frac{a}{2}\cos\omega t}\right)$$

$$V = -\frac{\partial\psi}{\partial t} = \frac{\mu I}{2\pi} b\left(\frac{a}{2}\omega\sin\omega t\right)\left[\left(d+\frac{a}{2}+\frac{a}{2}\cos\omega t\right)^{-1} + \left(d+\frac{a}{2}-\frac{a}{2}\cos\omega t\right)^{-1}\right]$$

$$= \frac{\mu I}{2\pi} b \frac{a\omega\left(d+\frac{a}{2}\right)}{\left(d+\frac{a}{2}\right)^2 - \left(\frac{a}{2}\cos\omega t\right)^2}\sin\omega t$$

∴ Induced emf is not a pure sinusoidal voltage. it is approximately sinusodial when $d \gg a$.

16.9 (a) From C to A, $\Delta B = 0.4\ \text{Wb/m}^2$

$$\therefore V = -\frac{\Delta\psi}{\Delta t} = \frac{-\Delta BA}{\Delta t} = -\frac{0.4\times 3\times 10^{-7}}{1\times 10^{-6}} = -0.12\ \text{V}$$

(b) From A to A', $\Delta B = 0.02\ \text{Wb/m}^2$

$$\therefore V = -\frac{\Delta\psi}{\Delta t} = \frac{-\Delta BA}{\Delta t} = -\frac{0.02\times 3\times 10^{-7}}{0.5\times 10^{-6}} = -0.012\ \text{V}$$

16.10 $L = \mu n^2\pi a^2 l$; $F = \frac{I^2}{2}\frac{dL}{da} = \frac{I^2}{2}2\mu n^2\pi a l = I^2\mu n^2\pi a l = (10)^2\times 4\pi\times 10^{-7}\times\left(\frac{100}{0.1}\right)^2\times\pi\times 0.01\times 0.1 = 0.395\ \text{N}$

16.11 $F = I^2\mu n^2\pi a l = (10\times 10^{-3})^2\times 1000\times 4\pi\times 10^{-7}\times\left(\frac{100}{0.1}\right)^2\times\pi\times 0.01\times 0.1 = 0.395\times 10^{-3}\ \text{N}$

16.12 From (16.34), $\langle P_L\rangle/l = (\pi a d\sigma)\frac{(\omega a\mu_0 H_0/2)^2}{1+(\omega\mu_0\sigma a d/2)^2}$, where $H_0 = nI = 200\times 10 = 2000$ and

$\pi a d\sigma = \pi\times 0.02\times 0.001\times 5.92\times 10^7 = 3719.6$,

$\omega a\mu_0 H_0/2 = 2\pi\times 10^3\times 0.02\times 4\pi\times 10^{-7}\times 2000/2 = 0.1579$,

$\omega\mu_0\sigma a d/2 = 2\pi\times 10^3\times 4\pi\times 10^{-7}\times 5.92\times 10^7\times 0.02\times 0.001/2 = 4.674$.

$\therefore \langle P_L\rangle/l = (3719.6)\times\frac{(0.1579)^2}{1+(4.674)^2} = 4.059\ \text{W/m}$

16.13 (a) $N_1 I_1 = N_2 I_2 \Rightarrow I_2 = \frac{N_1}{N_2} I_1 = \frac{100}{1000}\times 60 = 6\ \text{A}$

(b) From (16.37), $N_1 I_1 - N_2 I_2 = \Psi l/(\mu A) \Rightarrow 100\times 60 - 1000\times I_2 = \frac{Bl}{\mu} = \frac{0.3\times 0.4}{4\pi\times 10^{-7}\times 1100} = 86.81$

$\therefore I_2 = \frac{1}{1000}(100\times 60 - 86.81) = 5.91\ \text{A}$

(c) $N_1 I_1 = N_2 I_2$ is a good approximation of (16.37) for the material with large μ.

16.14 (a) $N_1 I_1 - N_2 I_2 = Hl$

$500 - 480 = H\times 0.4 \rightarrow H = 50 \rightarrow B = 2000\times 4\pi\times 10^{-7}\times 50 = 0.126\ \text{Wb/m}^2$ (clockwise)

(b) $|V_1| = N_1\omega\Psi$

$= 50\times 2\pi\times 60\times 0.126\times 4\times 10^{-4}$

$= 0.95\ \text{V}$

$|V_2| = \frac{N_2}{N_1}|V_1| = \frac{20}{50}\times 0.95 = 0.38\ \text{V}$

16.15 (a) $N_1 I_1 = N_2 I_2 \rightarrow I_2 = 150\times 60/450 = 0$ A

(b) $V_2/V_1 = N_2/N_1 \rightarrow V_2 = (450/60)\cos(120\pi t) = 7.5\cos(120\pi t)$

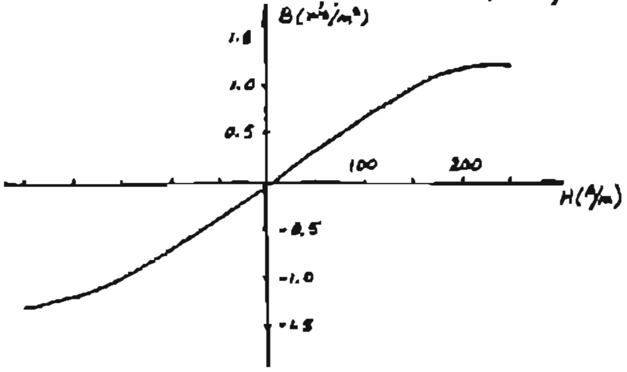
(c) $N_1 I_1 - N_2 I_2 = Hl = (B/\mu)l$

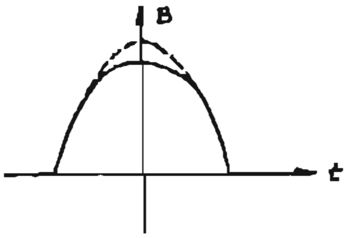
$$450 I_2 = 150\times 60 - 0.25\times 0.5/(1000\times 4\pi\times 10^{-7}).$$

$$I_2 = 19.78 \text{ A}$$

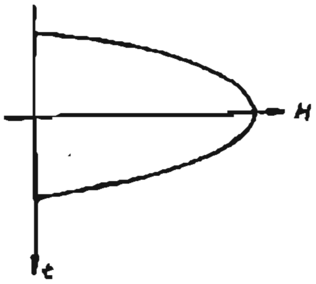
(d) $P = 90\times[4\times10^{-4}\times 0.5]\times 60 = 1.08$ W
(volume) (frequency)

16.16





The shape of $\Psi(t)$, or $V_1(t)$, will be distorted if $\hat{B}$ is too high.



16.17 Use (16.43), $W = (\Delta BH)\times(\text{volume})$

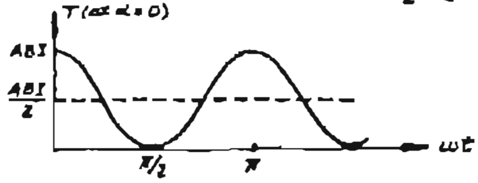
From Figure P16.7, $\Delta BH \doteq 0.2\times180\times4 = 144$ J/m³

$\therefore W = 144\times 2\pi\times 6\times10^{-4}\times 3\times10^{-7} = 1.629\times10^{-7}$ J

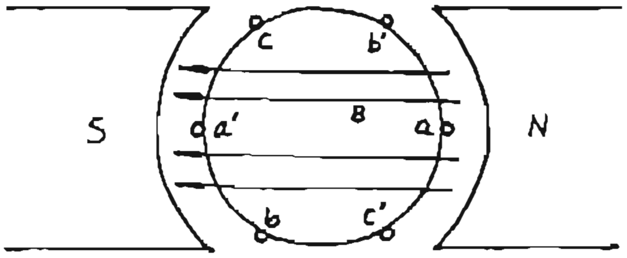
Since switched 1000 times per second, $P = 1000\times1.629\times10^{-7} = 0.163$ mW

16.18 From (16.50), $T = ABI\cos(\omega t)\cos(\omega t+\alpha) = \frac{ABI}{2}[\cos\alpha + \cos(2\omega t+\alpha)]$

$\langle T\rangle = \frac{ABI}{2}\cos\alpha$ and for $\alpha = 0$, $T = \frac{ABI}{2}[1+\cos(2\omega t)]$



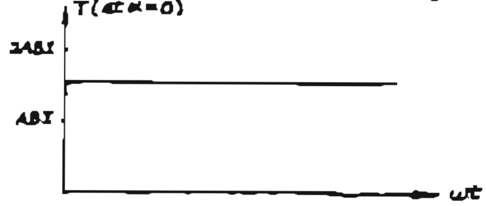
16.19



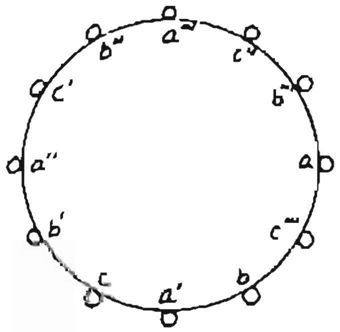
16.20 $T = \frac{ABI}{2}[3\cos\alpha + \cos(2\omega t+\alpha) + \cos(2\omega t+\alpha+240°) + \cos(2\omega t+\alpha+120°)$

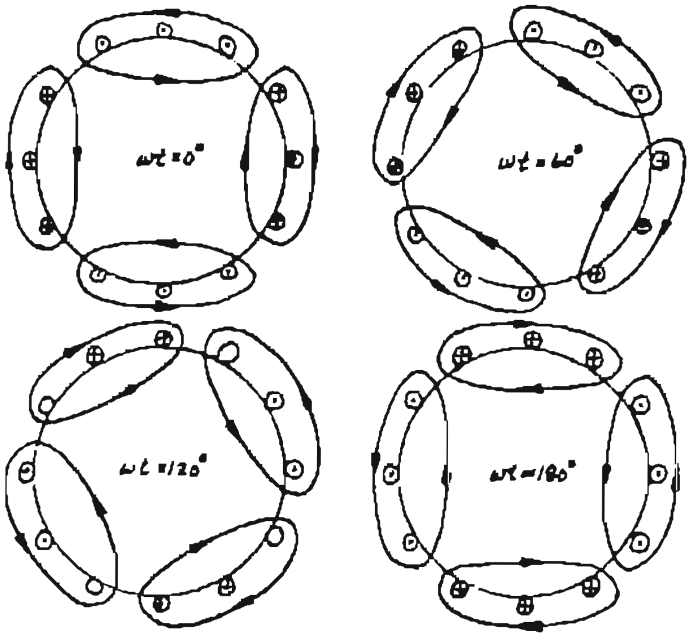
$= \frac{ABI}{2}[3\cos\alpha + 2\cos(120°)\cos(2\omega t+\alpha+120°) + \cos(2\omega t+\alpha+120°)]$

$= \frac{ABI}{2}[3\cos\alpha + \cos(2\omega t+\alpha+120°)(2\cos 120°+1)] = \frac{ABI}{2}(3\cos\alpha)$, independent of ωt.



16.21





16.22 From (16.67), $\sigma(1 - v_r/v_s) = kd/(s\mu_0 v_s)$ for maximum torque.

For larger σ, $(1 - v_r/v_s)$ should be smaller, or v_r/v_s should be closer to 1.

From Figure 16.21, when v_r/v_s close to 1, $\sigma_1 > \sigma_2 > \sigma_3$.

16.23 It becomes a generator.

16.24 $v = 250$ km/hr $= 250\times10^3/3600 = 69.44$ m/s

$l_t = 2\pi v/\omega = v/f = 69.44/60 = 1.157$ m

Appendix E Answers to Odd-Numbered Problems

Chapter 1

1.1 (a) $5 + j3$ **(b)** $11 + j$ **(c)** $-26 + j2$ **(d)** $-2.2 - j1.4$ **1.3** $\cos \omega t$; $\sin \omega t$; 1 **1.5** $\pm 2^{1/4} e^{j(\pi/8)}$ **1.7** Proof **1.9 (a)** $\sqrt{2} \cos\left(\omega t + \frac{\pi}{4}\right)$ **(b)** $4 \cos(\omega t + 0.8)$ **(c)** $3 \cos\left(\omega t + \frac{\pi}{2}\right) + 4 \cos(\omega t + 0.8)$ **1.11 (a)** $-6\hat{x} + 5\hat{y} + 2\hat{z}$ **(b)** $-10\hat{x} + 13\hat{y} - 4\hat{z}$ **(c)** -55 **(d)** $23\hat{x} + 22\hat{y} + 14\hat{z}$ **1.13** Proof **1.15** $\frac{1}{\sqrt{93}}(5\hat{x} - 8\hat{y} + 2\hat{z})$ **1.17** Proof omitted **1.19** Proof omitted **1.21** Proof omitted **1.23** $j\omega[(3 - j4)\hat{x} + 8(1 + j)\hat{z}]$ **1.25 (a)** $(-1 + j3)\hat{y} + (1 + j3)\hat{z}$ **(b)** $2\hat{x} + (1 - j)\hat{y} + (1 + j)\hat{z}$ **(c)** -5 **(d)** $4\hat{x} - (1 + j3)\hat{y} + (-1 + j3)\hat{z}$ **1.27** Sketch omitted

Chapter 2

2.1 $-6y\hat{x} - 3x^2\hat{y}$, $6z$ **2.3** Proof **2.5** Proof **2.7** No **2.9** $B(y,t) = 0.3(k/\omega) \cos(\omega t + ky)\hat{z}$ **2.11** $E_1 + E_2$, $H_1 + H_2$ $B_1 + B_2$ and $D_1 + D_2$, superposition theorem **2.13** Proof omitted **2.15** $\nabla \times \mathbf{E} = i\omega \mathbf{B}$, $\nabla \times \mathbf{H} = \mathbf{J} - i\omega \mathbf{D}$, $\nabla \cdot \mathbf{B} = 0$ and $\nabla \cdot \mathbf{D} = \rho_v$ **2.17** Proof omitted **2.19** $\hat{z}$, $\hat{z}$ **2.21** Proof omitted **2.23** $U_H/U_E = 1.13 \times 10^7$

Chapter 3

3.1 3.6×10^{-16} W/m^2 **3.3** 4×10^{26} W **3.5** 4.1×10^{13} km **3.7 (a)** rad/sec **(b)** m^{-1} **(c)** sec^{-1} **(d)** sec **(e)** m **3.9 (a)** 2.63 m **(b)** 0.704 m **3.11** Yes, $-\hat{z}$ direction, $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$, $-\frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}} E_0^2 \hat{z}$ **3.13** No, Maxwell's equations not satisfied **3.15 (a)** Right-hand circular polarization **(b)** Right-hand circular polarization **(c)** Left-hand elliptical polarization **(d)** Linear polarization **3.17** Proof omitted **3.19 (a)** 1 **(b)** 1 **(c)** 1.58 **(d)** 2.12 **3.21** 1.34×10^{-5} m, aluminum foil is about 9δ thick **3.23** 2.65 mW/m^2 **3.25 (a)** $E_x = e^{-0.5z} e^{-j0.5z}$ **(b)** $\mathbf{H} = \hat{y}(0.5 - j0.5) e^{-0.5z} e^{-j0.5z}$ **(c)** Sketch omitted **(d)** Sketch omitted **3.27** 0.6×10^{-6} m

Chapter 4

4.1 61.9° **4.3 (i)** c **(ii)** f **(iii)** b **(iv)** a **(v)** d **(vi)** e **4.5** Yes, circular shape, $0.3\,a^2$ on each surface (a is the length of each side of the cube.) **4.7** bevelled angle $- 35°$;
mirror making 70° with $\hat{z}$ axis; $\hat{x}$ polarized **4.9** $|E_{1x}| = \frac{1}{2}|E_0||1 - e^{-j25.3°} e^{jk_0 z}|$, $|E_{1z}| = \frac{\sqrt{3}}{2}|E_0||1 + e^{-j25.3°} e^{jk_0 z}|$, $|E_{2x}| = 0.5|E_0||1 - e^{-j25.3°}|e^{-\sqrt{80.75}k_0 z}$, $|E_{2z}| = \frac{\sqrt{3}}{160}|E_0||1 + e^{-j25.3°}|e^{-\sqrt{80.75}k_0 z}$ **4.11 (a)** $9\epsilon_0$ **(b)** 75 MHz

4.13 $\cos^{-1}\sqrt{\dfrac{\mu_1(\mu_2\epsilon_2 - \mu_1\epsilon_1)}{\epsilon_1(\mu_2^2 - \mu_1^2)}}$, $\cos^{-1}\sqrt{\dfrac{\epsilon_1(\mu_2\epsilon_2 - \mu_1\epsilon_1)}{\mu_1(\epsilon_2^2 - \epsilon_1^2)}}$
4.15 (a) 80 cm in front of the plate
(b) 2 V/m **4.17** 1.996 $|E_0|$ **4.19** $\mathbf{H}^i = \hat{\mathbf{y}}\, H_0 e^{-jkx\sin\theta - jkz\cos\theta}$,
$\mathbf{E}^i = (\hat{\mathbf{x}}\cos\theta - \hat{\mathbf{z}}\sin\theta)\, H_0\eta e^{-jkx\sin\theta - jkz\cos\theta}$, $\mathbf{H}^r = \hat{\mathbf{y}} H_0 e^{-jkx\sin\theta + jkz\cos\theta}$,
$\mathbf{E}^r = -(\hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta)\, H_0\eta e^{-jkx\sin\theta + jkz\cos\theta}$, where $\eta = \sqrt{\dfrac{\mu}{\epsilon}}$
4.21 $x = 0.87$ m, $y = 1.5$m, 12.04dB **4.23** Proof omitted

Chapter 5

5.1 Proof **5.3** 1.875 kHz **5.5** $\mathbf{E} = \hat{\mathbf{x}}\, E_0 e^{jkz}$, $\mathbf{H} = -\hat{\mathbf{y}}\,\dfrac{E_0}{\eta}\, e^{jkz}$, $\mathbf{J}_s = -\hat{\mathbf{z}}\,\dfrac{E_0}{\eta}\, e^{jkz}$ on lower plate, $\mathbf{J}_s = \hat{\mathbf{z}}\,\dfrac{E_0}{\eta}\, e^{jkz}$ on upper plate **5.7** 89.33 kW **5.9** Proof
5.11 5.26–10.52 GHz for 2.85 × 1.262 (cm) waveguide, 21.1–42.2 GHz for 0.711 × 0.355 (cm) waveguide **5.13** 1.318 MW
5.15 $E_y = E_1 \sin(\pi x/a)\, e^{jk_z z}$
$H_x = (E_1 k_z/\omega\mu)\sin(\pi x/a)\, e^{jk_z z}$
$H_z = (j\, E_1 \pi/\omega\mu a)\cos(\pi x/a)\, e^{jk_z z}$
where $k_z = [\omega^2\mu\epsilon - (\pi/a)^2]^{1/2}$
5.17 $\theta = \tan^{-1}(na/mb)$ **5.19** 5.83 GHz
5.21 Proof omitted **5.23** (0.866, 0.5, 2)
5.25 Proof omitted **5.27** $\mathbf{A} = 4.93\,\hat{\mathbf{x}} + 7.46\,\hat{\mathbf{y}} - 3\hat{\mathbf{z}}$ **5.29 (a)** $\mathbf{E}^i = \hat{\rho}\,\dfrac{V_1}{\rho} e^{-jk_1 z}$,
$\mathbf{H}^i = \hat{\phi}\,\dfrac{V_1}{\eta_1\rho}\, e^{-jk_1 z}$ **(b)** $V_0' = V_0(\eta_1 - \eta_0)/(\eta_1 + \eta_0)$, $V_1 = 2V_0\,\eta_1/(\eta_1 + \eta_0)$

Chapter 6

6.1 2000V **6.3** $(a_1/a_2) = (b_1/b_2) = \sqrt{1000}$ **6.5** $\mathbf{J}_s = \hat{\mathbf{z}}\,\dfrac{V_0}{\eta a}\, e^{-jkz}$, $I = \dfrac{2\pi V_0}{\eta}\, e^{-jkz}$
6.7 (a) $|V(z)| = 2|V_+||\cos kz|$, $|I(z)| = 2\dfrac{|V_+|}{Z_0}|\sin kz|$
(b) Sketch omitted **(c)** ∞ **6.9 (a)** 2.96 **(b)** $z = -0.35\lambda$ **(c)** 24.5%
6.11 0.342λ **6.13** $d = 0.25$ cm **6.15 (a)** 1.26 + j1.61 **(b)** 0.54 **(c)** 1
6.17 (a) 0.67 + j1.33 **(b)** 0.15 − j0.09 **6.19** Proof omitted **6.21** 14.2 kW
6.23 48.6% **6.25** Sketch omitted **6.27** Sketch omitted **6.29** Sketch omitted
6.31 Proof omitted

Chapter 7

7.1 (0.75, 0.433, 0.5) **7.3** Proof omitted **7.5** $\hat{\mathbf{x}}\cdot\hat{\theta} = \cos\theta\cos\phi$, $\hat{\mathbf{x}}\cdot\hat{\phi} = -\sin\phi$, $\hat{\mathbf{y}}\cdot\hat{\mathbf{r}} = \sin\theta\sin\phi$, $\hat{\mathbf{y}}\cdot\hat{\theta} = \cos\theta\sin\phi$, $\hat{\mathbf{y}}\cdot\hat{\phi} = \cos\phi$, $\hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = \cos\theta$, $\hat{\mathbf{z}}\cdot\hat{\theta} = -\sin\theta$, $\hat{\mathbf{z}}\cdot\hat{\phi} = 0$
7.7 Proof omitted **7.9** Yes, improved to 18% **7.11** $\mathbf{E} = (-\hat{\mathbf{y}})\,\dfrac{jkI\Delta z\eta e^{-jkx}}{8\pi x}$, linear **7.13 (a)** 0.314 V/m **(b)** 0.628 V/m **(c)** 6 V/m **7.15(a)** 1 **(b)** 1.5
(c) 1.64 **7.17** Six lobes; beam width = 19.2° along $\phi = 0$; beam width = 26.4° along $\phi = 41.8°$ **7.19** $D = 4$ **7.21** Sketch omitted **7.23 (a)** −90° **(b)** 6; 1.414; 0; 1.414 V/m **(c)** Sketch omitted **(d)** Sketch omitted

Chapter 8

8.1 $U_H/U_E \sim (kr)^2 << 1$ **8.3** When background is dark, one sees light scattered by the smoke particles. Blue light is scattered more strongly than red light. Against a bright background, one sees light passing through the smoke. The blue light gets scattered, and red and yellow lights suffer less scattering. **8.5** Proof omitted
8.7 360 km in radius **8.9** Train is moving toward the intersection. **8.11** Bandwidth = 59 kHz, 6.7 μs for 1 km resolution **8.13** Circular but opposite hand
8.15 No

Chapter 9

9.1 Exact: **(a)** 5.5302×10^{-10}V **(b)** 5.54244×10^{-12}V Approximate: **(a)** 5.5426×10^{-10}V **(b)** 5.54256×10^{-12}V **9.3** x = 1m plane, y = 1.5m plane **9.5** $(-\hat{r})\dfrac{1.44 \times 10^{-3}}{r^2}$ V/m **9.7** Proof omitted **9.9** Sketch omitted **9.11** **(a)** $\hat{\rho}\dfrac{\rho\ell}{40\pi\epsilon h}(0.0499)$ V/m **(b)** $\hat{\rho}\dfrac{\rho\ell}{40\pi\epsilon h}(0.05)$ V/m **(c)** 0.2% **9.13** **E** = 0 for $r < a$ and $b \leqslant r < c$, $\mathbf{E} = \hat{r}\dfrac{q}{4\pi\epsilon r^2}$ for $a \leqslant r < b$ and $r \geqslant c$ **9.15** $\dfrac{10^{-6}}{\epsilon r^2}[2 - e^{-r}(r^2 + 2r + 2)]$ **9.17** $\dfrac{10^{-6}}{\epsilon}(3e^{-1} - 1)$ Hint: $\int dre^{-r}(1 + r)/r^2 = -e^{-r}/r$ **9.19** 3 V, independent of path **9.21** **(a)** $\dfrac{q}{4\pi\epsilon c}$ **(b)** $\dfrac{q}{4\pi\epsilon c}$ **(c)** $\dfrac{q}{4\pi\epsilon}\left(\dfrac{1}{r} + \dfrac{1}{c} - \dfrac{1}{b}\right)$ **(d)** $\dfrac{q}{4\pi\epsilon}\left(\dfrac{1}{a} + \dfrac{1}{c} - \dfrac{1}{b}\right)$ **9.23** **(a)** 4.03×10^{-9} C **(b)** 330 kV **(c)** 0.4 mA

Chapter 10

10.1 $-\dfrac{4}{9}q$ **10.3** 1.8×10^{-5} N (attractive) **10.5** **(a)** $r(2.4\pi \times 10^{4})$ V/m **(b)** $a \times 1.2\pi \times 10^{-8}$N (repulsive) **(c)** No **10.7** 0.36 mm **10.9** z = ± 3.14 cm
10.11 **(a)** $v_0 = 1.874 \times 10^7$ m/s, $v_{0z} = 1.867 \times 10^7$ m/s, $v_{0x} = -0.163 \times 10^7$ m/s
(b) $x(t) = (8.78 \times 10^{14})t^2 - (0.163 \times 10^7)t$ m, $z(t) = (1.867 \times 10^7)t$ m **(c)** $x = -3.52 \times 10^{-4}$ m, $z = 3 \times 10^{-2}$ m
10.13 Proof omitted **10.15** 5.3×10^{-10} farad **10.17** **(a)** $\dfrac{A}{d}\left(\dfrac{\epsilon_1 + \epsilon_2}{2}\right)$
(b) $Q_1 = \dfrac{\epsilon_1 Q}{\epsilon_1 + \epsilon_2}$, $Q_2 = \dfrac{\epsilon_2 Q}{\epsilon_1 + \epsilon_2}$ **10.19** $2\pi \Big/ \left[\dfrac{1}{\epsilon_1}\ln(c/a) + \dfrac{1}{\epsilon_2}\ln(b/c)\right]$
10.21 4.97×10^{-6} J **10.23** **(a)** $Q^2S/2A\epsilon_0$ **(b)** $-Q^2/2A\epsilon_0$ (attractive)

Chapter 11

11.1 $\Phi_3 = \Phi_1 + \Phi_2$ **11.3** $\Phi = V\ln(b/\rho)/\ln(b/a)$ **11.5** $\Phi(\theta) = V_0 \ln\left(\tan\dfrac{\theta}{2}\right)\Big/\ln\left(\tan\dfrac{\theta_1}{2}\right)$, $\rho_s = -V_0\epsilon_0\Big/\left[r\ln\left(\tan\dfrac{\theta_1}{2}\right)\sin\theta\right]$ **11.7** 12.3 V **11.9** Sketch omitted
11.11 17.5 $\mu\mu$ F/m **11.13** $-q(d^2 - a^2)/[4\pi a(a^2 + d^2 - 2ad\cos\theta)^{3/2}]$
11.15 $\dfrac{q}{4\pi\epsilon}\left(\dfrac{1}{R_1} - \dfrac{a}{dR_2}\right) + \dfrac{q(a/d) + q_0}{4\pi\epsilon r}$, where $R_1 = (r^2 + d^2 - 2rd\cos\theta)^{1/2}$, $R_2 = \left(r^2 + \dfrac{a^4}{d^2} - 2\dfrac{ra^2}{d}\cos\theta\right)^{1/2}$ **11.17** $(\rho_\ell)^2/[2\pi\epsilon(d - b)]$ where $d = a^2/b$, attractive
11.19 $(d/a)q^2/[4\pi\epsilon(d - b)^2]$, where $d = a^2/b$, attractive **11.21** $200\sin(2\pi x/a)\sinh(2\pi y/a)/\sinh(2\pi b/a)$

11.23 $(400/\pi)\displaystyle\sum_{n\text{-odd}}^{\infty}\frac{1}{n}\left\{\frac{\sin(n\pi x/a)\sinh(n\pi y/a)}{\sinh(n\pi b/a)} + \frac{\sin(n\pi y/b)\sinh(n\pi x/b)}{\sinh(n\pi a/b)}\right\}$

11.25
(a) $\dfrac{V_0}{\left(\dfrac{1}{a} - \dfrac{1}{b}\right)}\left(\dfrac{1}{r} - \dfrac{1}{b}\right)$ **(b)** $\dfrac{V_0}{\left(\dfrac{1}{a} - \dfrac{1}{b}\right)r^2}$ **(c)** $D_{r1} = \dfrac{V_0\epsilon_1}{\left(\dfrac{1}{a} - \dfrac{1}{b}\right)r^2}$ **(d)** $Q = \dfrac{2\pi V_0}{\left(\dfrac{1}{a} - \dfrac{1}{b}\right)}(\epsilon_1 + \epsilon_2)$

$D_{r2} = \dfrac{V_0\epsilon_2}{\left(\dfrac{1}{a} - \dfrac{1}{b}\right)r^2}$ $C = \dfrac{2\pi(\epsilon_1 + \epsilon_2)}{\left(\dfrac{1}{a} - \dfrac{1}{b}\right)}$

Chapter 12

12.1 $C = A(\epsilon_1 + \epsilon_2)/2d$, $G = A(\sigma_1 + \sigma_2)/2d$; C in parallel with G **12.3** **(a)** $I\ln(a/\rho)/(2\pi\sigma_1\ell)$ **(b)** $I\ln(a/b)/(2\pi\sigma_1\ell) + I\ln(b/\rho)/(2\pi\sigma_2\ell)$ **(c)** $\dfrac{I}{2\pi\ell}\left[\dfrac{\ln(b/a)}{\sigma_1} + \dfrac{\ln(c/b)}{\sigma_2}\right]$

12.5 0.92×10^{-3} mho/m **12.7** 106 V

12.9 $\left(\dfrac{\sigma_2}{\sigma_1 + \sigma_2}\right) \times 100\%$ **12.11** 12.3 Ω-m **12.13** Sketch omitted

12.15 $\rho_v = 600 \left\{\dfrac{1}{6} - \dfrac{50}{d[(y + 6 - r_0 y/d)^2 + (70 - 70 r_0/d)^2]^{1/2}} + \dfrac{50}{d[(y + 6)^2 + (70)^2]^{1/2}}\right\},$
where $d = [(70)^2 + y^2]^{1/2}$, $r_0 = (50)^2/[(70)^2 + y^2]^{1/2}$

Chapter 13

13.1 $\hat{z}\, 2\sqrt{2}\, I/(\pi b)$ **13.3** $\boldsymbol{H} = (-\hat{x})Jy$ for $|y| < \dfrac{d}{2}$, $\boldsymbol{H} = (-\hat{x})J(d/2)$ for $y > d/2$, $\boldsymbol{H} = \hat{x}\, J(d/2)$ for $y < (-d/2)$ **13.5 (a)** 0 **(b)** $I(\rho^2 - a^2)/[2\pi\rho(b^2 - a^2)]$ **(c)** $I/2\pi\rho$
13.7 (a) $(-\hat{z})Ia(dy)/[4\pi(a^2 + y^2)^{3/2}]$ **(b)** $(-\hat{z})I/(4\pi a)$ **(c)** 0 **(d)** $(-\hat{z})I/(4\pi a)$
13.9 $z_0 = \pm\, 0.27a$

13.11
$$H_{AB} = \hat{z}\,\frac{aI}{4\pi}\int_{-a}^{a}\frac{dx}{[a^2 + (b - x)^2]^{3/2}}$$

$$H_{BC} = \hat{z}\,\frac{-I(b - a)}{4\pi}\int_{-a}^{a}\frac{dx}{[y^2 + (b - a)^2]^{3/2}}$$

$$H_{CD} = H_{AB}$$

$$H_{DA} = \hat{z}\,\frac{I(b + a)}{4\pi}\int_{-a}^{a}\frac{dx}{[y^2 + (b + a)^2]^{3/2}}$$

13.13 2.8 MHz **13.15 (a)** x = 0.04 m, z = −0.00725 m **(b)** −20.5° **13.17** 1.33×10^{-3} N/m (repulsive) **13.19 (a)** Loop should be placed horizontally or vertically in the east-west direction **(b)** 7.85×10^{-3} N-m **(c)** Vertically in north-south direction
13.21 (a) $\boldsymbol{H}_\rho = \dfrac{I\rho}{2\pi a^2}$ **(b)** $U_H = \dfrac{\mu I^2}{16\pi}$ **(c)** 5×10^{-8} H/m **13.23** 2.74×10^{-6} Joule/meter **13.25** $\dfrac{\mu_0}{2\pi}\left\{\dfrac{1}{4} + \ln(b/a) + \dfrac{c^4 \ln(c/b)}{(c^2 - b^2)^2} - \dfrac{c^2}{c^2 - b^2} + \dfrac{c^2 + b^2}{4(c^2 - b^2)}\right\}$

Chapter 14

14.1 Sketch **14.3** 0.21 weber/m², 100 A/m **14.5** 0.71 weber/m² **14.7** $x_1y_1 = 1$, $x_1y_2 = 0$, $x_3y_3 = 1$, $x_3y_4 = 0$, $x_1y_2 = 1$, $x_2y_2 = 1$, $x_4y_2 = 1$ **14.9** 4.233A **14.11** 8.32A **14.13** 0.29 weber/m²

Chapter 15

15.1 $\mathbf{E}^{(0)} = 0$, $\mathbf{H}^{(0)} = -\hat{y} I_0 \cos(\omega t)/w$; $\mathbf{E}^{(1)} = -\hat{x} I_0 \omega\mu z \sin(\omega t)/w$, $\mathbf{H}^{(1)} = 0$; $\mathbf{E}^{(2)} = 0$, $\mathbf{H}^{(2)} = \hat{y}\, I_0 \omega^2 \epsilon\mu \left(\dfrac{1}{2}\right) z^2 \cos(\omega t)/w$; $\mathbf{E}^{(3)} = \hat{x}\, I_0 \omega^3 \mu^2 \epsilon \left(\dfrac{1}{2}\right)\left(\dfrac{1}{3}\right) z^3 \sin(\omega t)/w$; $\mathbf{H}^{(3)} = 0$
15.3 $U_H^{(1)} = \dfrac{\mu}{2}\omega^2\epsilon^2 V_0^2 \left(\dfrac{1}{a^2}\right) \sin^2(\omega t) \left(\dfrac{1}{3}\right) aw\ell^3$, $U_E^{(1)} = 0$ **15.5** $U_E^{(0)} = 0$, $U_H^{(0)} = \dfrac{\mu}{2}\dfrac{I_0^2}{w^2}\cos^2(\omega t)(wa\ell)$
15.7 $E_\rho^{(0)} = V_0 \cos(\omega t)/[\rho \ln(b/a)]$, $I^{(0)} = 2\pi V_0 \sigma \ell \cos(\omega t)/\ln(b/a)$, $Q^{(0)} = 2\pi V_0 \epsilon \ell \cos(\omega t)/\ln(b/a)$, $I^{(1)} = -2\pi V_0 \epsilon \ell \omega \sin(\omega t)/\ln(b/a)$ **15.9** 11.1 sec

Chapter 16

16.1 101 A **16.3** $V_1 = 0.25$ V, $V_2 = -0.15$ V, $V_3 = -0.25$ V **16.5 (a)** 0.125 mA (clockwise) **(b)** $V_1 = -0.25$ V; $V_2 = -0.625$ V **16.7** 58.9 mV
16.9 (a) −0.12 V **(b)** −0.012 V **16.11** 0.395×10^{-3} N **16.13 (a)** 6 A **(b)** 5.91 A
16.15 (a) 20 A **(b)** 330 cos (120πt) **(c)** 19.78 A **(d)** 1.08 W **16.17** 0.16 mW
16.19 Sketch omitted **16.21** Sketch omitted **16.23** It becomes a generator.